CSI5180. Machine Learning for Bioinformatics Applications

Decision Trees

Marcel Turcotte

Version November 6, 2019

Preamble

Preamble

Decision Trees

In this lecture, we discuss decision trees. One of their main advantages is interpretability. Also, they naturally work with a mixture of data (feature) types. When used in the context of ensemble learning, their performance is generally excellent.

General objective :

Explain what decision trees are, how they are built, and how they can be used to classify data.

Learning objectives

- **Explain** what decision trees are.
- **Decribe** the algorithm to construct a decision tree.
- Discuss the concept of purity.
- **Explain** how to use decision trees to classify new examples.

Reading:

- Carl Kingsford and Steven L Salzberg. What are decision trees? Nat Biotechnol 26(9):10113, Sep 2008.
- Pierre Geurts, Alexandre Irrthum, and Louis Wehenkel. Supervised learning with decision tree-based methods in computational and systems biology. *Mol Biosyst* 5(12):1593605, Dec 2009.

Plan

1. Preamble

2. Introduction

- 3. What is a decision tree?
- 4. Construction
- 5. Limitation
- 6. Random Forest
- 7. Applications

8. Prologue

Introduction

Decision trees are **supervised** learning algorithms.

- Decision trees are supervised learning algorithms.
- Tasks:

- Decision trees are supervised learning algorithms.
- Tasks:
 - **Regression:** *y_i* is a real value (not discussed here);

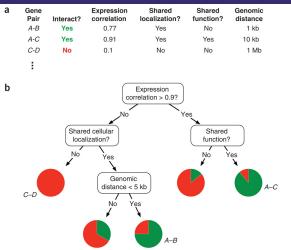
- Decision trees are supervised learning algorithms.
- Tasks:
 - **Regression:** *y_i* is a real value (not discussed here);
 - **Classification:** *y_i* is a class.

- Decision trees are supervised learning algorithms.
- Tasks:
 - **Regression:** *y_i* is a real value (not discussed here);
 - **Classification:** *y_i* is a class.
- At the heart of **Random Forest** algorithms, which are some of the **best** algorithms when the **number of available examples is low**.

- Decision trees are supervised learning algorithms.
- Tasks:
 - **Regression:** *y_i* is a real value (not discussed here);
 - **Classification:** *y_i* is a class.
- At the heart of **Random Forest** algorithms, which are some of the **best** algorithms when the **number of available examples is low**.
- Handles a mixture of categorical and real-valued features, as well as missing values, depending on the implementation (Scikit-Learn's implementation does not handle the missing values).

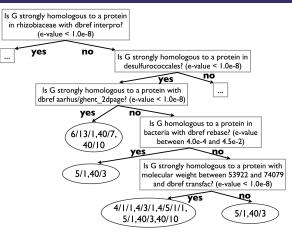
- Decision trees are supervised learning algorithms.
- Tasks:
 - **Regression:** *y_i* is a real value (not discussed here);
 - **Classification:** *y_i* is a class.
- At the heart of **Random Forest** algorithms, which are some of the **best** algorithms when the **number of available examples is low**.
- Handles a mixture of categorical and real-valued features, as well as missing values, depending on the implementation (Scikit-Learn's implementation does not handle the missing values).
- The resulting **model** is easily **interpretable** by humans!

Interpretable models



C. Kingsford and S. L. Salzberg. What are decision trees? Nat Biotechnol 26(9):10113, 2008.

Interpretable models



Schietgat, L. et al. Predicting gene function using hierarchical multi-label decision tree ensembles. BMC bioinformatics 11, 14 (2010).

A decision tree is a hierarchical structure (directed acyclic graph) used to make decisions (classification, regression).

- A decision tree is a hierarchical structure (directed acyclic graph) used to make decisions (classification, regression).
- Each **internal node** is a (binary) test for some feature *j*.

- A decision tree is a hierarchical structure (directed acyclic graph) used to make decisions (classification, regression).
- Each **internal node** is a (binary) test for some feature *j*.
 - E.g. is the **expression of a given gene in the given sample** higher than some threshold.

- A decision tree is a hierarchical structure (directed acyclic graph) used to make decisions (classification, regression).
- Each **internal node** is a (binary) test for some feature *j*.
 - E.g. is the **expression of a given gene in the given sample** higher than some threshold.
- Leaves are decision nodes.

- A decision tree is a hierarchical structure (directed acyclic graph) used to make decisions (classification, regression).
- Each **internal node** is a (binary) test for some feature *j*.
 - E.g. is the **expression of a given gene in the given sample** higher than some threshold.
- Leaves are decision nodes.
- The "structure" of the tree is learnt from the training data.

- A decision tree is a hierarchical structure (directed acyclic graph) used to make decisions (classification, regression).
- Each **internal node** is a (binary) test for some feature *j*.
 - E.g. is the **expression of a given gene in the given sample** higher than some threshold.
- Leaves are decision nodes.
- The "structure" of the tree is learnt from the training data.

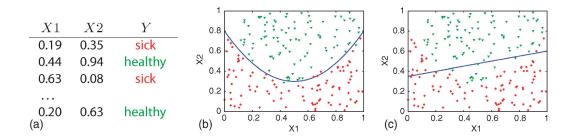
- A decision tree is a hierarchical structure (directed acyclic graph) used to make decisions (classification, regression).
- Each **internal node** is a (binary) test for some feature *j*.
 - E.g. is the **expression of a given gene in the given sample** higher than some threshold.
- Leaves are decision nodes.
- The "structure" of the tree is learnt from the training data.

 \Rightarrow More complex representations are possible (for instance, ID3 allows for more than two children).

It suffices to start at the root of the tree, answering a series of binary questions, until reaching a leave, the value found in that leave is the label of that example.

- It suffices to start at the root of the tree, answering a series of binary questions, until reaching a leave, the value found in that leave is the label of that example.
 - In some cases (algorithms), a leave contains a probability distribution the proportion of the training data belonging to class k, for all values of k, in this leaf.

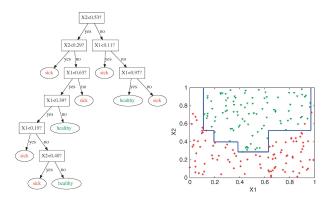
Simple decision boundary



Logistic Regression has a simple decision boundary

Source: Geurts, P., Irrthum, A. & Wehenkel, L. Supervised learning with decision tree-based methods in computational and systems biology. *Mol Biosyst* **5** 15931605 (2009).

Complex decision boundary



Decision trees can produce a complex and irregular decision boundary

Source: Geurts, P., Irrthum, A. & Wehenkel, L. Supervised learning with decision tree-based methods in computational and systems biology. *Mol Biosyst* **5** 15931605 (2009).

Construction

Incremental process: starting with an empty tree, nodes are added one by one guided by the training data until the data is perfectly classified (or some other criteria applies, e.g. maximum depth).

- Incremental process: starting with an empty tree, nodes are added one by one guided by the training data until the data is perfectly classified (or some other criteria applies, e.g. maximum depth).
- Consider the case of **creating the first node**.

- Incremental process: starting with an empty tree, nodes are added one by one guided by the training data until the data is perfectly classified (or some other criteria applies, e.g. maximum depth).
- Consider the case of **creating the first node**.
 - It makes sense to consider all D features to create a test. For each feature, one can consider several thresholds (based on the observed values in our training set).

- Incremental process: starting with an empty tree, nodes are added one by one guided by the training data until the data is perfectly classified (or some other criteria applies, e.g. maximum depth).
- Consider the case of **creating the first node**.
 - It makes sense to consider all D features to create a test. For each feature, one can consider several thresholds (based on the observed values in our training set).
 - > What would be good criteria to select the best candidate?

- Incremental process: starting with an empty tree, nodes are added one by one guided by the training data until the data is perfectly classified (or some other criteria applies, e.g. maximum depth).
- Consider the case of **creating the first node**.
 - It makes sense to consider all D features to create a test. For each feature, one can consider several thresholds (based on the observed values in our training set).
 - > What would be good criteria to select the best candidate?
 - Heterogeneity (impurity) and homogeneity of the resulting sets.

- Incremental process: starting with an empty tree, nodes are added one by one guided by the training data until the data is perfectly classified (or some other criteria applies, e.g. maximum depth).
- Consider the case of **creating the first node**.
 - It makes sense to consider all D features to create a test. For each feature, one can consider several thresholds (based on the observed values in our training set).
 - > What would be good criteria to select the best candidate?
 - Heterogeneity (impurity) and homogeneity of the resulting sets.
 - Ideally, each one of the two resulting sets would contain data from a single class!

- Incremental process: starting with an empty tree, nodes are added one by one guided by the training data until the data is perfectly classified (or some other criteria applies, e.g. maximum depth).
- Consider the case of **creating the first node**.
 - It makes sense to consider all D features to create a test. For each feature, one can consider several thresholds (based on the observed values in our training set).
 - > What would be good criteria to select the best candidate?
 - Heterogeneity (impurity) and homogeneity of the resulting sets.
 - Ideally, each one of the two resulting sets would contain data from a single class!
 - **Entropy** and **Geni index** are two popular choices.

Objective function for sklearn.tree.DecisionTreeClassifier (CART):

$$J(k, t_k) = rac{m_{ ext{left}}}{m} G_{ ext{left}} + rac{m_{ ext{right}}}{m} G_{ ext{right}}$$

Objective function for sklearn.tree.DecisionTreeClassifier (CART):

$$J(k, t_k) = rac{m_{ ext{left}}}{m} G_{ ext{left}} + rac{m_{ ext{right}}}{m} G_{ ext{right}}$$

The cost of partitioning the data using **feature** k and **threshold** t_k .

Objective function for sklearn.tree.DecisionTreeClassifier (CART):

$$J(k, t_k) = rac{m_{ ext{left}}}{m} G_{ ext{left}} + rac{m_{ ext{right}}}{m} G_{ ext{right}}$$

- The cost of partitioning the data using **feature** k and **threshold** t_k .
- *m*_{left} and *m*_{right} is the **number of examples** in the **left** and **right** subsets, respectively, and *m* is the number of examples before splitting the data.

Objective function for sklearn.tree.DecisionTreeClassifier (CART):

$$J(k, t_k) = rac{m_{ ext{left}}}{m} G_{ ext{left}} + rac{m_{ ext{right}}}{m} G_{ ext{right}}$$

- The cost of partitioning the data using **feature** k and **threshold** t_k .
- *m*_{left} and *m*_{right} is the **number of examples** in the **left** and **right** subsets, respectively, and *m* is the number of examples before splitting the data.
- Gleft and Gright is the impurity of the left and right subsets, respectively.

Gini index (default)

$$G_i = 1 - \sum_{k=1}^n p_{i,k}^2$$

Gini index (default)

$$G_i = 1 - \sum_{k=1}^n p_{i,k}^2$$

p_{*i*,*k*} is the proportion of the examples from this class *k* in the node *i*.

Gini index (default)

$$G_i = 1 - \sum_{k=1}^n p_{i,k}^2$$

*p*_{*i*,*k*} is the proportion of the examples from this class *k* in the node *i*. **Examples**:

1

Gini index (default)

$$G_i = 1 - \sum_{k=1}^n p_{i,k}^2$$

 $p_{i,k}$ is the proportion of the examples from this class k in the node i.

>
$$1 - (0/100)^2 + (100/100)^2 = 0$$
 (pure)

Gini index (default)

$$G_i = 1 - \sum_{k=1}^n p_{i,k}^2$$

 $p_{i,k}$ is the proportion of the examples from this class k in the node i.

>
$$1 - (0/100)^2 + (100/100)^2 = 0$$
 (pure)

$$1 - (25/100)^2 + (75/100)^2 = 0.375$$

Gini index (default)

$$G_i = 1 - \sum_{k=1}^n p_{i,k}^2$$

 $p_{i,k}$ is the proportion of the examples from this class k in the node i.

>
$$1 - (0/100)^2 + (100/100)^2 = 0$$
 (pure)

$$1 - (25/100)^2 + (75/100)^2 = 0.375$$

$$1 - (50/100)^2 + (50/100)^2 = 0.5$$

Gini index (default)

$$G_i = 1 - \sum_{k=1}^n p_{i,k}^2$$

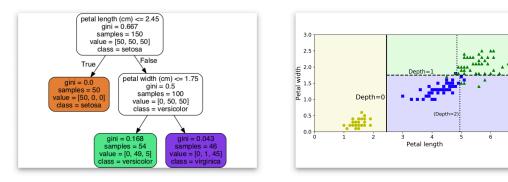
 $p_{i,k}$ is the proportion of the examples from this class k in the node i.

>
$$1 - (0/100)^2 + (100/100)^2 = 0$$
 (pure)
> $1 - (25/100)^2 + (75/100)^2 = 0.375$
> $1 - (50/100)^2 + (50/100)^2 = 0.5$
> $1 - 10 \times (10/100)^2 = 0.9$

Gini index (default)

$$G_i = 1 - \sum_{k=1}^n p_{i,k}^2$$

 $p_{i,k}$ is the proportion of the examples from this class k in the node i.



Source: [6] Figures 6.1 and 6.2

7

Given a set of training examples, let p_k be proportion of these examples belonging to the class C_k for k ∈ {1, 2, ..., m} (a probability distribution).

- Given a set of training examples, let p_k be proportion of these examples belonging to the class C_k for k ∈ {1, 2, ..., m} (a probability distribution).
- **Entropy** is defined as follows:

$$H = -\sum_{k=1}^m p_k \log_2 p_k$$

- Given a set of training examples, let p_k be proportion of these examples belonging to the class C_k for k ∈ {1, 2, ..., m} (a probability distribution).
- Entropy is defined as follows:

$$\mathcal{H} = -\sum_{k=1}^{m} p_k \log_2 p_k$$

Entropy is maximum when the events are all equiprobable. The maximum value is log₂ m.

- Given a set of training examples, let p_k be proportion of these examples belonging to the class C_k for k ∈ {1, 2, ..., m} (a probability distribution).
- Entropy is defined as follows:

$$\mathcal{H} = -\sum_{k=1}^m p_k \log_2 p_k$$

- Entropy is maximum when the events are all equiprobable. The maximum value is log₂ m.
- The entropy approaches **0** as the probability of one class, p_k , approches **1**.

- Given a set of training examples, let p_k be proportion of these examples belonging to the class C_k for k ∈ {1, 2, ..., m} (a probability distribution).
- Entropy is defined as follows:

$$\mathcal{H} = -\sum_{k=1}^m p_k \log_2 p_k$$

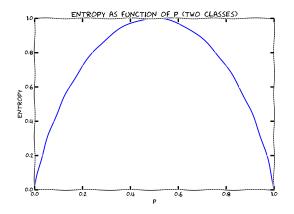
- Entropy is maximum when the events are all equiprobable. The maximum value is log₂ m.
- The entropy approaches **0** as the probability of one class, p_k , approches **1**.
- Entropy and Gini produce similar results:

- Given a set of training examples, let p_k be proportion of these examples belonging to the class C_k for k ∈ {1, 2, ..., m} (a probability distribution).
- Entropy is defined as follows:

$$\mathcal{H} = -\sum_{k=1}^m p_k \log_2 p_k$$

- Entropy is maximum when the events are all equiprobable. The maximum value is log₂ m.
- The entropy approaches **0** as the probability of one class, p_k , approches **1**.
- Entropy and Gini produce similar results:
 - https://sebastianraschka.com/faq/docs/decision-tree-binary.html

Entropy (uncertainty) for two classes



Consider a **probability space** with two **outcomes**.

$$H = -[(1-p)\log_2(1-p) + p\log_2 p]$$

Entropy (uncertainty) for two classes

```
import matplotlib
import matplotlib.pyplot as plt
import numpy as np
def lg2(v):
    return np.where(v != 0.0, np.log2(v), 0.0)
with plt.xkcd():
    p = np.arange(0.0, 1.0, 0.001)
    h = -((1-p) * |g_2(1-p) + p*|g_2(p))
    fig, ax = plt.subplots()
    ax.plot(p, h)
    ax.set(xlabel='p', ylabel='Entropy', title='Entropy as...')
    ax.grid()
    plt.show()
```

All the examples in a given node belong to the same class.

- All the examples in a given node belong to the same class.
- Depth of the tree would exceed **max_depth**.

- All the examples in a given node belong to the same class.
- Depth of the tree would exceed **max_depth**.
- Number of examples in the node is **min_sample_split** or less.

- All the examples in a given node belong to the same class.
- Depth of the tree would exceed **max_depth**.
- Number of examples in the node is **min_sample_split** or less.
- None of the splits decreases impurity sufficiently (min_impurity_decrease).

- All the examples in a given node belong to the same class.
- Depth of the tree would exceed **max_depth**.
- Number of examples in the node is **min_sample_split** or less.
- None of the splits decreases impurity sufficiently (min_impurity_decrease).
- See documentation for other criteria.

Possibly creates large trees

- Possibly creates large trees
 - > Challenge for interpretation

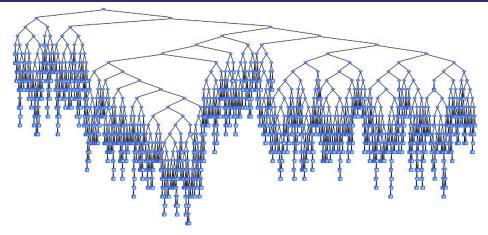
- Possibly creates large trees
 - Challenge for interpretation
 - Overfitting

- Possibly creates large trees
 - Challenge for interpretation
 - Overfitting
- **Greedy algorithm**, no guarantee to find the optimal tree.

- Possibly creates large trees
 - Challenge for interpretation
 - Overfitting
- **Greedy algorithm**, no guarantee to find the optimal tree.
- Small changes to the data set, produces vastly different trees.

- Possibly creates large trees
 - Challenge for interpretation
 - Overfitting
- **Greedy algorithm**, no guarantee to find the optimal tree.
- Small changes to the data set, produces vastly different trees.
- The decision boundaries are orthogonal, which makes then sensitive to rotations (this can be alleviated by first running PCA [Principal Component Analysis], then applying decision trees on the transformed data).

Large trees



Stiglic, G., Kocbek, S., Pernek, I. & Kokol, P. Comprehensive Decision Tree Models in Bioinformatics. *PLoS ONE* **7**, (2012).

Limiting the **maximum depth** of the tree is a form of **regularization**.

Regularization

- Limiting the **maximum depth** of the tree is a form of **regularization**.
 - Likewise, the values for the other parameters, such as min_impurity_decrease, can be determined using a validation set.

Limiting the **maximum depth** of the tree is a form of **regularization**.

- Likewise, the values for the other parameters, such as min_impurity_decrease, can be determined using a validation set.
- Another regularization technique for decision trees is known as pruning.

Regularization

- Limiting the **maximum depth** of the tree is a form of **regularization**.
 - Likewise, the values for the other parameters, such as min_impurity_decrease, can be determined using a validation set.
- Another regularization technique for decision trees is known as pruning.
 - In a bottom-up fashion, nodes are removed if this reduces the classification error on the validation set.

Regularization

Limiting the **maximum depth** of the tree is a form of **regularization**.

- Likewise, the values for the other parameters, such as min_impurity_decrease, can be determined using a validation set.
- Another regularization technique for decision trees is known as pruning.
 - In a bottom-up fashion, nodes are removed if this reduces the classification error on the validation set.
- https:

//scikit-learn.org/dev/auto_examples/tree/plot_cost_complexity_pruning.html

Implementations

- **ID3** (Iterative Dichotomiser 3) **ID3**, **C4.5**, **C5.0** by Ross Quinlan.
- **CART** (Classification And Regression Tree) by Leo Breiman *et al.*
- sklearn.tree.DecisionTreeClassifier
 - https://scikit-learn.org/stable/modules/tree.html

```
from sklearn.tree import DecisionTreeClassifier
# ...
clf = tree.DecisionTreeClassifier()
clf = clf.fit(X, y)
tree.plot_tree(clf)
# ...
clf.predict(X_new)
```

Ensemble methods will be discussed with greater details later.

- **Ensemble** methods will be discussed with greater details later.
- "Although single decision trees can be excellent classifiers, increased accuracy often can be achieved by combining the results of a collection of decision trees." [3]

- **Ensemble** methods will be discussed with greater details later.
- Although single decision trees can be excellent classifiers, increased accuracy often can be achieved by combining the results of a collection of decision trees." [3]
- A Random Forest is a collection of decision trees.

- **Ensemble** methods will be discussed with greater details later.
- Although single decision trees can be excellent classifiers, increased accuracy often can be achieved by combining the results of a collection of decision trees." [3]
- A Random Forest is a collection of decision trees.
 - Strategies to build a collection of trees:

- **Ensemble** methods will be discussed with greater details later.
- Although single decision trees can be excellent classifiers, increased accuracy often can be achieved by combining the results of a collection of decision trees." [3]
- A Random Forest is a collection of decision trees.
 - Strategies to build a collection of trees:
 - Creating new data sets using a sampling with replacement procedure (boostrap sampling);

- **Ensemble** methods will be discussed with greater details later.
- "Although single decision trees can be excellent classifiers, increased accuracy often can be achieved by combining the results of a collection of decision trees." [3]
- A Random Forest is a collection of decision trees.
 - Strategies to build a collection of trees:
 - Creating new data sets using a sampling with replacement procedure (boostrap sampling);
 - Using a random subset of the features for splitting (typically the square root of the total number of features);

- **Ensemble** methods will be discussed with greater details later.
- "Although single decision trees can be excellent classifiers, increased accuracy often can be achieved by combining the results of a collection of decision trees." [3]
- A Random Forest is a collection of decision trees.
 - Strategies to build a collection of trees:
 - Creating new data sets using a sampling with replacement procedure (boostrap sampling);
 - Using a random subset of the features for splitting (typically the square root of the total number of features);
 - Taking advantage of the **stochastic nature** of the procedure to build trees.

- **Ensemble** methods will be discussed with greater details later.
- "Although single decision trees can be excellent classifiers, increased accuracy often can be achieved by combining the results of a collection of decision trees." [3]
- A Random Forest is a collection of decision trees.
 - Strategies to build a collection of trees:
 - Creating new data sets using a sampling with replacement procedure (boostrap sampling);
 - Using a random subset of the features for splitting (typically the square root of the total number of features);
 - Taking advantage of the **stochastic nature** of the procedure to build trees.
 - Prediction: the most common prediction (majority vote) amongst all the trees (the information can be used as an indication of the strength of the prediction).

Other ensemble learning techniques, such as bagging, pasting, boosting, and stacking will be discussed later.

Applications

- "Synthetic sick and lethal (SSL) genetic interactions between genes A and B occur when the organism exhibits poor growth (or death) when both A and B are knocked out but not when either A or B is disabled individually."
 [3]
- Determine the exon-intron structure of eukaryotic genes (gene finders). [3]
- In the study of gene expression profilling. [3]
- Cancer classification [4].





Decision trees can solve both tasks, **classification** and **regression**.



- Decision trees can solve both tasks, classification and regression.
- Handle a mixture of data (feature) types, real and categorical.



- Decision trees can solve both tasks, classification and regression.
- Handle a mixture of data (feature) types, real and categorical.
- Interpretation of the resulting model is high



- Decision trees can solve both tasks, classification and regression.
- Handle a **mixture of data (feature) types**, real and categorical.
- Interpretation of the resulting model is high
- Stability might be an issue, which can be alleviated when coupled with ensemble learning.

Hidden Markov Models

References

- Gregor Stiglic, Simon Kocbek, Igor Pernek, and Peter Kokol. Comprehensive decision tree models in bioinformatics. *PLoS One*, 7(3):e33812, 2012.
- Pierre Geurts, Alexandre Irrthum, and Louis Wehenkel. Supervised learning with decision tree-based methods in computational and systems biology. *Mol Biosyst*, 5(12):1593–605, Dec 2009.
- Carl Kingsford and Steven L Salzberg.
 What are decision trees?
 Nat Biotechnol, 26(9):1011–3, Sep 2008.
- Dongsheng Che, Qi Liu, Khaled Rasheed, and Xiuping Tao. Decision tree and ensemble learning algorithms with their applications in bioinformatics.

Adv Exp Med Biol, 696:191–9, 2011.

References

Leander Schietgat, Celine Vens, Jan Struyf, Hendrik Blockeel, Dragi Kocev, and Saso Dzeroski.

Predicting gene function using hierarchical multi-label decision tree ensembles. *BMC Bioinformatics*, 11:2, Jan 2010.

Aurélien Géron.

Hands-on Machine Learning with Scikit-Learn, Keras, and TensorFlow. O'Reilly Media, 2nd edition, 2019.

Andriy Burkov.

The Hundred-Page Machine Learning Book. Andriy Burkov, 2019.



Marcel Turcotte

Marcel.Turcotte@uOttawa.ca

School of Electrical Engineering and Computer Science (EECS) University of Ottawa