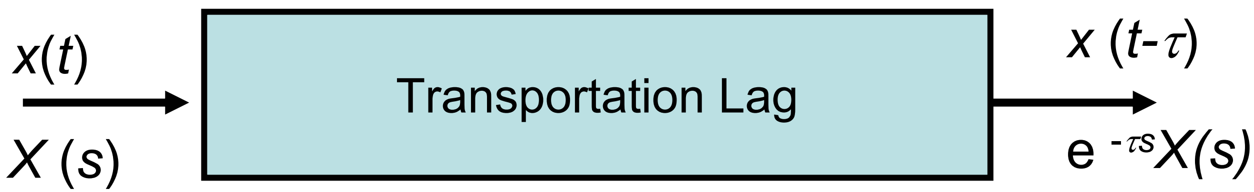
The Transportation Lag

The transportation lag is the delay between the time an input signal is applied to a system and the time the system reacts to that input signal. Transportation lags are common in industrial applications. They are often called "**dead time**".



Example

$$G_p(s) = \frac{10e^{-\tau s}}{s(s+4)}; \tau = 0.5s;$$ For unity feedback we wish to design a compensator that satifies : $t_p \le 0.5\text{s}; PO \le 10\%$

Choose $\xi = 1/\sqrt{2}$.Now satisfy the angle condition

$$\angle e^{-\tau s} - \angle s - \angle s + 4 = -180^{\text{o}}.\ s = -a + ja$$

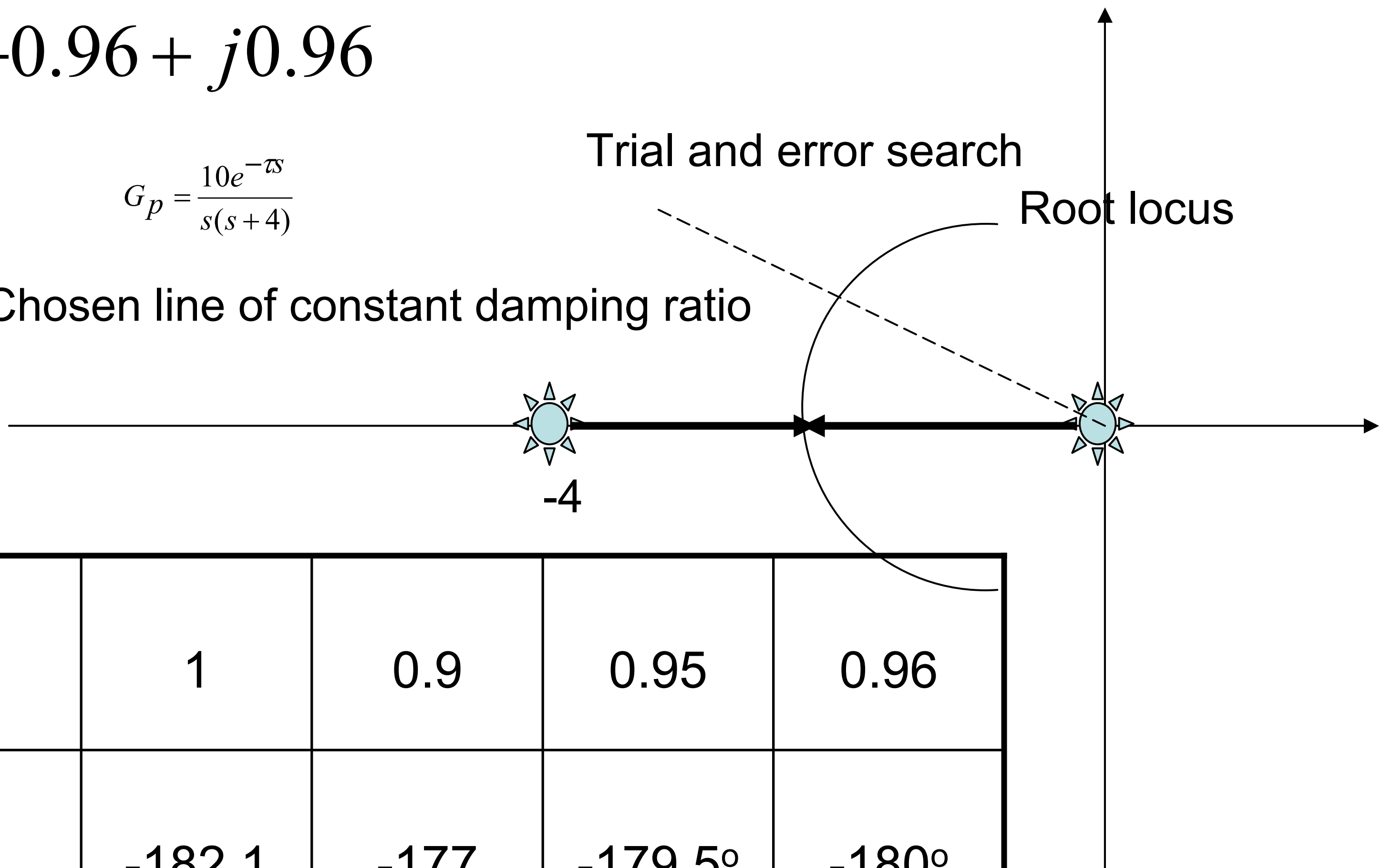
$$e^{-\tau s} = e^{-(-a+ja)\tau} = e^{a\tau}e^{-ja\tau} = e^{a\tau}\angle -a\tau$$

$$\angle e^{-(-a+ja)\tau} = -a\tau \times \frac{180^{\text{o}}}{\pi} = -a\tau \times 57.3^{\text{o}}$$

$$-a\tau \times 57.3^{\text{o}} - \angle s - \angle s + 4 = -180^{\text{o}}$$

$$s = -0.96 + j0.96$$

$$G_p = \frac{10e^{-\tau s}}{s(s+4)}$$



a	1	0.9	0.95	0.96
$\angle G_p$	-182.1	-177	-179.5^o	-180^o

$$K = \left[\frac{|s| \times |s+4|}{10 \times \left| e^{-\tau s} \right|} \right]_{s=0.96+j0.96} = 0.268$$

$$e^{(0.96)(0.5)} = 1.616.$$

This is the contribution from the transportation to the gain.

In this case, we used simple gain compensator. The design procedure is no more difficult if we add lead, lag, or PID compensation.

Example

$G_p(s) = \dfrac{10e^{-\tau s}}{s(s+1)}$; $\tau = 0.5s$. Design a compensator with the specifications

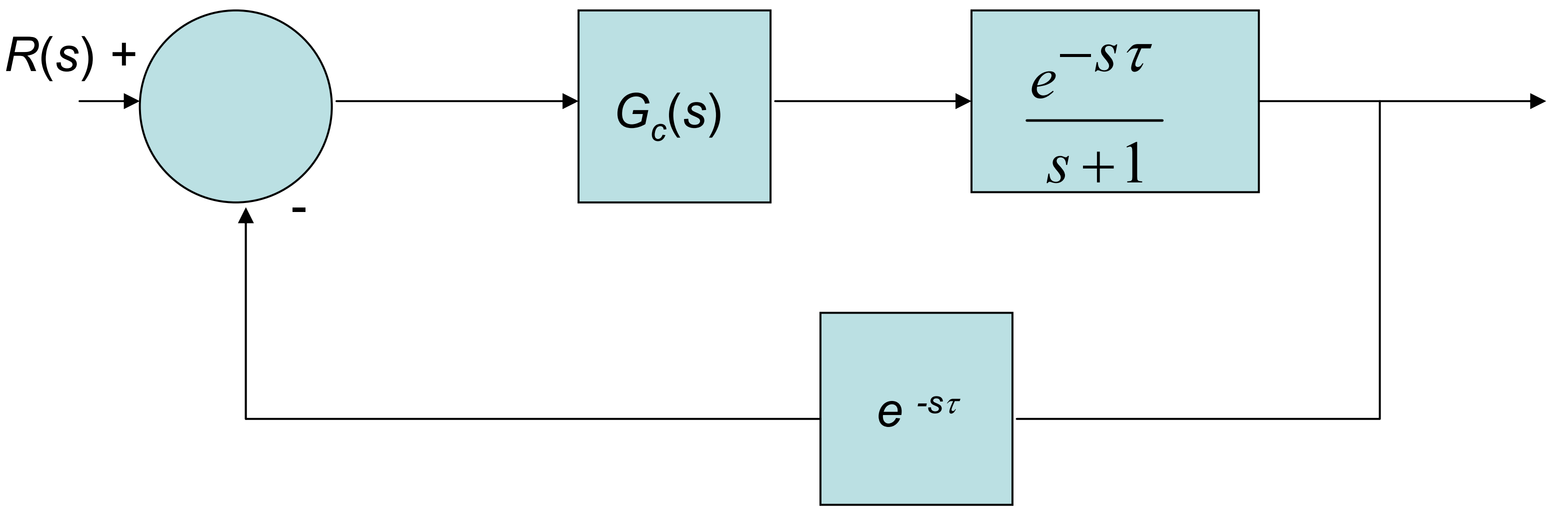
Crossover frequency $\omega_c = 1$ rad/s; phase margin $\phi_m \geq 55^{\circ}$;

Velocity error constant K_v as large as possible.

Example

- Suppose we wish to put a lunar rover on the moon that has a simple robot arm. We want to control the rover and its robot arm with TV camera from earth. An operator sitting in a console manuevers the rover and the robot arm using a joystick. It takes 1.27 s for a signal sent from earth to reach the rover on the moon. It takes another 1.27 s for the TV image to be transmitted back to earth. Our goal is to compensate this system so that it behaves in a stable fashion. The specifications are:
- $e_{ss} = 5\%$
- A quick response

Block Diagram of Lunar Robot Control



Digital System with Transportation Lag

Consider a system with $G_p(s) = \dfrac{4e^{-\tau s}}{s(s+4)}$ with $\tau = 0.5$s.

Design $G_c(s)$ accounting for the negative phase contribution of the zero-order hold and that of the transportation lag. Transfer $G_c(s)$ to the z domain. The performance specifications are:

Sampling rate = 50 Hz; $K_v \rangle 100$; $\omega_c \geq 1$ rad/s

Phase margin of at least 50^{o}.

The negative phase contribution of the zero-order hold is

$-\dfrac{\omega T}{2}$ rad = -28 ωT degree. The negative phase contribution of the transportation lag = $-\omega\tau$ rad = -57.3 $\omega\tau$ degree.

Phase-Lead Design Using the Bode Plot

- Evaluate the uncompensated system phase margin when the error constants are satisfied.
- Allowing for a small amount of safety (for example, negative phase contribution of the transportation lag = -57$\omega\tau$) determine the necessary additional phase lead ϕ_m.
- Evaluate α from Equation (10.11).
- Evaluate 10 log α and determine the frequency where the uncompensated magnitude curve is equal to - 10 log α dB. Calculate ω_m which is equal to ω_c.
- Calculate the pole and zero
- Draw the compensated frequency response and raise the gain of the amplifier in order to account for the attenuation.

$$p = \omega_m \sqrt{\alpha} \qquad z = p / \alpha$$