CSI 4105 Design and Analysis of Algorithms II — Fall 2003
Computer Science — University of Ottawa

Homework Assignment #2 (100 points, weight 10%)
Due: November 13 (10:00 a.m.) in lecture.

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1. (20 points) **DNF-SAT is polynomial-time solvable**
A formula is in disjunctive normal form (DNF) if it is a disjunction (or's) of clauses that are conjunctions (and's) of literals. Note that no assumption is made about the number of variables in each clause.
Example: $\Phi = (x_1 \wedge x_2 \wedge \neg x_1 \wedge x_3) \vee (x_2 \wedge x_3 \wedge \neg x_4) \vee (x_1 \wedge \neg x_1)$ is satisfiable.
Prove that the problem of determining the satisfiability of a boolean formula in disjunctive normal form is polynomial-time solvable. (To prove that you have to provide an algorithm to solve the problem and show that it runs in polynomial time).

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2. (30 points) **0-1 Integer Programming Problem is NP-complete**
Given an integer $m \times n$ matrix A and an integer m-vector b, the **0-1 integer programming problem** asks whether there is an integer n-vector x with elemens in the set $\{0, 1\}$ such that $Ax \geq b$.
Note that the problem is simply asking whether the following system of equations have a solution with each $x_j \in \{0, 1\}$, $1 \leq i \leq n$:

$$\begin{aligned}
a_{1,1}x_1 + a_{1,2}x_2 + \ldots a_{1,n}x_n &\geq b_1, \\
a_{2,1}x_1 + a_{2,2}x_2 + \ldots a_{2,n}x_n &\geq b_2, \\
\vdots \quad &\quad \vdots \\
a_{m,1}x_1 + a_{m,2}x_2 + \ldots a_{m,n}x_n &\geq b_m.
\end{aligned}$$

1. Define the language associated to the 0-1 integer programming problem.
2. Prove that the 0-1 integer programming problem is NP-complete.
   Hint: reduce from 3-Cnf-Sat

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3. (30 points) HamPath **is NP-complete**
A **hamiltonian path** in a graph is a simple path that visits every vertex exactly once. Let HamPath= $\{< G, u, v >$: there is a hamiltonian path from u to v$\}$.
Show that HamPath is NP-complete. Hint: reduce from HamCycle.

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4. (20 points) HamPath **for directed acyclic graphs is polynomial-time solvable**
A directed graph is called **acyclic** if it does not contain a directed cycle. Show that the hamiltonian path problem can be solved in polynomial time on directed acyclic graphs. This means that you need to show that the language
HamPathDirAcycl= $\{< G, u, v >$: G is a directed acyclic graph and there is a hamiltonian path from u to v$\}$ is in P.
Give an efficient algorithm for the decision problem and analyse its complexity.
Hint: Review Dijkstra's algorithm for shortest path and note that it works with negative weights for directed acyclic graphs.
