CSI 4105 Design and Analysis of Algorithms II — Fall 2001
Computer Science — University of Ottawa

Homework Assignment #2: Graph Colouring (100 points, weight 10%)
Due: Thursday, November 15, at 10:00 a.m. (in lecture)

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1. (70 points) **Graph colouring: problem 34-3 page 1019-1020 of the textbook**

34-3-a (10 points), 34-3-b (10 points), 34-3-c (10 points),
34-3-extra (10 points) Give two examples to illustrate the reduction from 3-CNF-SAT to graph colouring. One for a satisfiable 3-CNF formula and one for a non-satisfiable 3-CNF formula. Examples should be original (no similarities expected within the class). Choose examples that show that you have understood the reduction. Indeed, you may try on your own many more examples, since this will help you with parts d-f.
34-3-d (10 points), 34-3-e (10 points), 34-3-f (10 points).

2. (30 points) **Application of graph colouring: exam timetabling**

a. A modeling exercise (10 points)

Consider the problem of scheduling exams. An exam for course A and course B cannot be scheduled into the same time slot if there is some student taking both courses A and B; in this case we say that there is a conflict between courses A and B. The problem we are interested in solving is the one of determining the minimum number of time slots so that we are able to accommodate all exams without scheduling conflicting courses into the same time slot.
This problem can be modeled as a graph colouring problem. Consider an undirected graph in which vertices correspond to courses, and there is an edge connecting two vertices if and only if there is a conflict between their corresponding courses. Colours are associated to time slots.
Exercise: (limit 1 page) Invent an example (10 nodes) of a timetabling problem and show its model as a graph colouring problem (explain). Find a colouring of the graph and show the corresponding exam timetable.
Examples invented by different students should not be similar to each other. You will be marked for clarity of presentation and ability to illustrate well the general model (very simple graphs like one with no edges may be easy to explain, but don't illustrate well the general case!)

b. Some practical problems (20 points)
Suppose your boss asked you to program a polynomial-time algorithm to solve the exam timetabling problem. Since you have studied in CSI 4105 (assignment#2) that this is equivalent to a NP-complete problem, you tell your boss that he is asking you to solve one of the most challenging problems in modern Mathematics (you may even have to mention the $1 million prize one could get).
But your boss doesn't give up that easily. He has also taken CSI4105 in the past, and knows that sometimes there are special cases of hard problems that can be solved efficiently. So, he gives you a list of questions based on data collected on his particular graphs. Based on the given data, you should try to answer all the questions you can. There may be cases in which you simply have not enough information to answer (in these cases explain why the given information is not sufficient). There are some situations in which your boss gave you contradictory information (point them out precisely and give convincing explanations).
All questions are based on data in the following table:

University (graph name)	maximum degree of vertices:	cliques were found of sizes:	extra info
OttawaU	10	1,2,3	there are 100 courses
Carleton	3	$3, 4, 5, 6$	there are 100 courses
UBC	5	4	it is raining there
UofT	47	$1, 2, \ldots, 45$	there are 3 groups of vertices containing 10, 50 and 30 vertices with no edges between different groups
Berkeley	9	10	there are 200 courses

The questions you have to answer are: (individual/original answers sought!)

- Ottawa U: can we schedule the exams in 15 time slots? What is the minimum number of time slots you can guarantee to be sufficient? Justify.
- Give a lower bound and an upper bound on the minimum number of time slots required to schedule Carleton exams. Give the best possible bounds. Justify.
- Give a lower bound and an upper bound on the minimum number of time slots required to schedule UBC exams. Give the best possible bounds. Justify.
- Give a lower bound and an upper bound on the minimum number of time slots required to schedule UofT exams. Give the best possible bounds. Justify.
- For Berkeley you should answer: Can we schedule the exams using 7 time slots? Can we schedule the exams using 9 time slots? Can we schedule the exams using 10 time slots? Justify.