CSI2131, Winter 2004
Lab 11 Solution

Question 1

```
   +------+        +--------+
 0 |   ---|------->| 2, 6   | A
   +------+        +--------+
 1 |   ---|------->| 3, 7   | B
   +------+        +--------+
```

Insert 10: 10 in binary: 1010

- Bucket pointed by 0 is full.
- Depth of bucket A is 1 and depth of directory is 1. Since equal, must double the directory.

```
   +------+
00 |   ---|---+    +--------+
   +------+   +-->| 2, 6   | A
01 |   ---|---+    +--------+
   +------+
10 |   ---|---+    +--------+
   +------+   +-->| 3, 7   | B
11 |   ---|---+    +--------+
   +------+
```

- Now split bucket A.
- 2: 0010
- 6: 0110

```
   +------+        +--------+
00 |   ---|------->|        | A
   +------+        +--------+
01 |   ---|------->| 2, 6   | A1
   +------+        +--------+
10 |   ---|---+    +--------+
   +------+   +-->| 3, 7   | B
11 |   ---|---+    +--------+
   +------+
```

- Recursive call to insert 10.
- 10: 1010

- Bucket pointed by 01 is full.
- Depth of bucket A1 is 2 = depth of the directory. So must double the size.

```
    +------+
000 |  ---|---+   +-------+
    +------+   +-->|       | A
001 |  ---|---+   +-------+
    +------+
010 |  ---|---+   +-------+
    +------+   +-->| 2, 6  | A1
011 |  ---|---+   +-------+
    +------+
100 |  ---|---+
    +------+   |
101 |  ---|---+   +-------+
    +------+   +-->| 3, 7  | B
110 |  ---|---+   +-------+
    +------+   +
111 |  ---|---+
    +------+
```

- Now Split bucket A1.

```
    +------+
000 |  ---|---+   +-------+
    +------+   +-->|       | A
001 |  ---|---+   +-------+
    +------+      +-------+
010 |  ---|------>| 2     | A1
    +------+      +-------+
011 |  ---|------>| 6     | A2
    +------+      +-------+
100 |  ---|---+
    +------+   |
101 |  ---|---+   +-------+
    +------+   +-->| 3, 7  | B
110 |  ---|---+   +-------+
    +------+   +
111 |  ---|---+
    +------+
```

- 10: 1010
- Now bucket pointed by 010 is not full. Just place 10 there.

```
    +------+
000 |  ---|---+   +--------+
    +------+   +-->|        | A
001 |  ---|---+   +--------+
    +------+       +--------+
010 |  ---|------>| 2,10   | A1
    +------+       +--------+
011 |  ---|------>| 6      | A2
    +------+       +--------+
100 |  ---|---+
    +------+   |
101 |  ---|---+   +--------+
    +------+   +-->| 3, 7   | B
110 |  ---|---+   +--------+
    +------+   +
111 |  ---|---+
    +------+
```

Insert 15: 15 in binary: 1<u>111</u>

- Bucket pointed by 111 is full.
- Depth of bucket B is 1 ¡ depth of directory which is 3. So no need of doubling directory.
- Split bucket B.
- 3: <u>110</u>
- 7: <u>111</u>

```
    +------+
000 |  ---|---+   +--------+
    +------+   +-->|        | A
001 |  ---|---+   +--------+
    +------+       +--------+
010 |  ---|------>| 2,10   | A1
    +------+       +--------+
011 |  ---|------>| 6      | A2
    +------+       +--------+
100 |  ---|---+   +--------+
    +------+   +-->|        | B
101 |  ---|---+   +--------+
    +------+
110 |  ---|---+   +--------+
    +------+   +-->| 3, 7   | B1
111 |  ---|---+   +--------+
    +------+
```

- Recursive call to insert 15.
- 15: 1<u>111</u>
- Bucket pointed by 111(B1) is full.
- Depth of bucket B1 is 2 ¡ depth of the directory which is 3. So no need to doubling directory.
- Now, split B1.

```
    +------+
000 |   ---|---+   +--------+
    +------+   +-->|        | A
001 |   ---|---+   +--------+
    +------+       +--------+
010 |   ---|------>| 2,10   | A1
    +------+       +--------+
011 |   ---|------>| 6      | A2
    +------+       +--------+
100 |   ---|---+   +--------+
    +------+   +-->|        | B
101 |   ---|---+   +--------+
    +------+       +--------+
110 |   ---|------>| 3      | B1
    +------+       +--------+
111 |   ---|------>| 7      | B2
    +------+       +--------+
```

- Recursive call to insert 15 will find that there is space in bucket pointed by 111.

```
    +------+
000 |   ---|---+   +--------+
    +------+   +-->|        | A
001 |   ---|---+   +--------+
    +------+       +--------+
010 |   ---|------>| 2,10   | A1
    +------+       +--------+
011 |   ---|------>| 6      | A2
    +------+       +--------+
100 |   ---|---+   +--------+
    +------+   +-->|        | B
101 |   ---|---+   +--------+
    +------+       +--------+
110 |   ---|------>| 3      | B1
    +------+       +--------+
111 |   ---|------>| 7,15   | B2
    +------+       +--------+
```

Insert 4, 8:

- This is the summary, please do it one at a time.

```
    +------+
000 |   ---|---+   +--------+
    +------+   +-->| 4, 8   | A
001 |   ---|---+   +--------+
    +------+       +--------+
010 |   ---|------>| 2,10   | A1
    +------+       +--------+
011 |   ---|------>| 6      | A2
    +------+       +--------+
100 |   ---|---+   +--------+
    +------+   +-->|        | B
101 |   ---|---+   +--------+
    +------+       +--------+
110 |   ---|------>| 3      | B1
    +------+       +--------+
111 |   ---|------>| 7,15   | B2
    +------+       +--------+
```

- After insert 12 (1100) (split A).

```
    +------+       +--------+
000 |   ---|------>| 8      | A
    +------+       +--------+
001 |   ---|------>| 4,12   | A3
    +------+       +--------+
010 |   ---|------>| 2,10   | A1
    +------+       +--------+
011 |   ---|------>| 6      | A2
    +------+       +--------+
100 |   ---|---+   +--------+
    +------+   +-->|        | B
101 |   ---|---+   +--------+
    +------+       +--------+
110 |   ---|------>| 3      | B1
    +------+       +--------+
111 |   ---|------>| 7,15   | B2
    +------+       +--------+
```

Delete 3:

- Remove from B1.
- Run TryCombine(B1)
- 110 points to B1.
- 111 points to B2(buddy bucket).
- Combine B1 and B2.

```
    +------+       +--------+
000 |   ---|------>| 8      | A
    +------+       +--------+
001 |   ---|------>| 4,12   | A3
    +------+       +--------+
010 |   ---|------>| 2,10   | A1
    +------+       +--------+
011 |   ---|------>| 6      | A2
    +------+       +--------+
100 |   ---|---+   +--------+
    +------+   +-->|        | B
101 |   ---|---+   +--------+
    +------+
110 |   ---|---+   +--------+
    +------+   +-->| 7,15   | B1
111 |   ---|---+   +--------+
    +------+
```

- Cannot collapse directory.
- After delete 10.
- Run TryCombine(A1).
- 010 points to A1.
- 011 points to A2 (buddy bucket).
- Combine A1 and A2.

```
    +------+        +--------+
000 |   ---|------->| 8      | A
    +------+        +--------+
001 |   ---|------->| 4,12   | A3
    +------+        +--------+
010 |   ---|---+    +--------+
    +------+   +-->| 2, 6   | A1
011 |   ---|---+    +--------+
    +------+
100 |   ---|---+    +--------+
    +------+   +-->|        | B
101 |   ---|---+    +--------+
    +------+
110 |   ---|---+    +--------+
    +------+   +-->| 7,15   | B1
111 |   ---|---+    +--------+
    +------+
```

- Cannot collapse directory.

Delete 12:

- Remove from A3.
- Run TryCombine(A3).
- 001 points to A3.
- 000 points to A (buddy bucket).
- Combine A3 and A.

```
    +------+
000 |  ---|---+   +-------+
    +------+   +-->| 4, 8  | A3
001 |  ---|---+   +-------+
    +------+
010 |  ---|---+   +-------+
    +------+   +-->| 2, 6  | A1
011 |  ---|---+   +-------+
    +------+
100 |  ---|---+   +-------+
    +------+   +-->|       | B
101 |  ---|---+   +-------+
    +------+
110 |  ---|---+   +-------+
    +------+   +-->| 7,15  | B1
111 |  ---|---+   +-------+
    +------+
```

- This time we can collapse the directory.

```
    +------+       +-------+
 00 |  ---|------>| 4, 8  | A3
    +------+       +-------+
 01 |  ---|------>| 2, 6  | A1
    +------+       +-------+
 10 |  ---|------>|       | B
    +------+       +-------+
 11 |  ---|------>| 7,15  | B1
    +------+       +-------+
```

- Try Combine(B1) is called recursively.

```
    +------+       +-------+
 00 |  ---|------>| 4, 8  | A3
    +------+       +-------+
 01 |  ---|------>| 2, 6  | A1
    +------+       +-------+
 10 |  ---|---+   +-------+
    +------+   +-->| 7,15  | B1
 11 |  ---|---+   +-------+
    +------+
```

Delete 2:

```
   +------+        +--------+
00 |   ---|------->| 4, 8   | A3
   +------+        +--------+
01 |   ---|------->| 6      | A1
   +------+        +--------+
10 |   ---|---+    +--------+
   +------+   +-->| 7,15   | B1
11 |   ---|---+    +--------+
   +------+
```

Delete 6:

```
   +------+        +--------+
00 |   ---|------->| 4, 8   | A3
   +------+        +--------+
01 |   ---|------->|        | A1
   +------+        +--------+
10 |   ---|---+    +--------+
   +------+   +-->| 7,15   | B1
11 |   ---|---+    +--------+
   +------+
```

- Try Combine(A1) with buddy bucket A3.

```
   +------+
00 |   ---|---+    +--------+
   +------+   +-->| 4, 8   | A1
01 |   ---|---+    +--------+
   +------+
10 |   ---|---+    +--------+
   +------+   +-->| 7,15   | B1
11 |   ---|---+    +--------+
   +------+
```

- Now we can collapse the directory.

```
   +------+        +--------+
 0 |   ---|------->| 4, 8   | A1
   +------+        +--------+
 1 |   ---|------->| 7, 15 | B1
   +------+        +--------+
```

Question 2

A) Hash('PAL') $= (2 \times 80 + 65 + 3 \times 76)\ mod\ 6$
$= 453\ mod\ 6 = 3$

Hash('LAP') $= (2 \times 76 + 65 + 3 \times 80)\ mod\ 6$
$= 457\ mod\ 6 = 1$

B) As follows:

0	SET
1	LAV
2	PET
3	VAL
4	PAT
5	MAP

C) As follows:

Key	**Home Address**	**Search Length**
VAL	3	1
LAV	1	1
MAP	5	1
PAT	3	2
PET	1	2
SET	1	6

Average search length $= \frac{1+1+1+2+2+6}{6} = \frac{13}{6} \cong 2.17$

Question 3

A) As follows:

0	CAT	-1
1	PET	2
2	SET	-1
3	PAT	4
4	SAT	0
5	MAP	-1

B) As follows:

Key	**Home Address**	**Search Length**
MAP	5	1
PAT	3	1
PET	1	1
SET	1	2
SAT	3	2
CAT	3	3

Average search length $= \frac{1+1+1+2+2+3}{6} = \frac{10}{6} \cong 1.67$

C) As follows:

Hashed File

0		
1	PET	0
2		
3	PAT	1
4		
5	MAP	-1

Overflow Area

0	SET	-1
1	SAT	2
2	CAT	-1