Hashing

Today: Chapters 11.1-11.4.

Motivation

The main motivation for Hashing is improving searching time. Below we show how the search time for Hashing compares to the one for other methods:

- Sequential search: $O(N)$
- B Trees and B+ trees: $O(\log_k N)$
- Hashing: $O(1)$

What is Hashing ?

A **Hash Function** is a function $h(k)$ that transforms a key into an address. An address space is chosen before hand. For example, we may decide the file will have 1,000 available addresses.

If U is the set of all possible keys, the hash function is from U to {0,1,...,999}, that is

$$h : U \longrightarrow \{0, 1, ..., 999\}$$

Example :

k	ASCII code for the first 2 letters	product	h(k) = product mod 1,000
BALL	66, 65	66.65=4,290	290
LOWELL	76, 79	76.79=6,004	004
TREE	84, 82	84.82=6,888	888

RRN	FILE
000	
001	
⋮	⋮
004	LOWELL
⋮	⋮
290	BALL
⋮	⋮
888	TREE
⋮	⋮
999	

- There is no obvious connection between the key and the location (randomizing).
- Two different keys may be sent to the same address generating a **Collision**

Can you give an example of collision for the hash function in the previous example ?

LOWELL, LOCK, OLIVER, and any word with first two letters L and O will be mapped to the same address:

$$h(\text{LOWELL}) = h(\text{LOCK}) = h(\text{OLIVER}) = 4.$$

These keys are called **synonyms**. The address "4" is said to be the **home address** of any of these keys.

Avoiding collisions is extremely difficult (remember the birthday paradox discussed in class), so we need techniques for dealing with it.

Ways of reducing collisions:

1. **Spread out the records** by choosing a good hash function.
2. **Use extra memory**, i.e. increase the size of the address space (Ex: reserve 5,000 available addresses rather than 1,000).
3. **Put more than one record at a single address** (use of buckets).

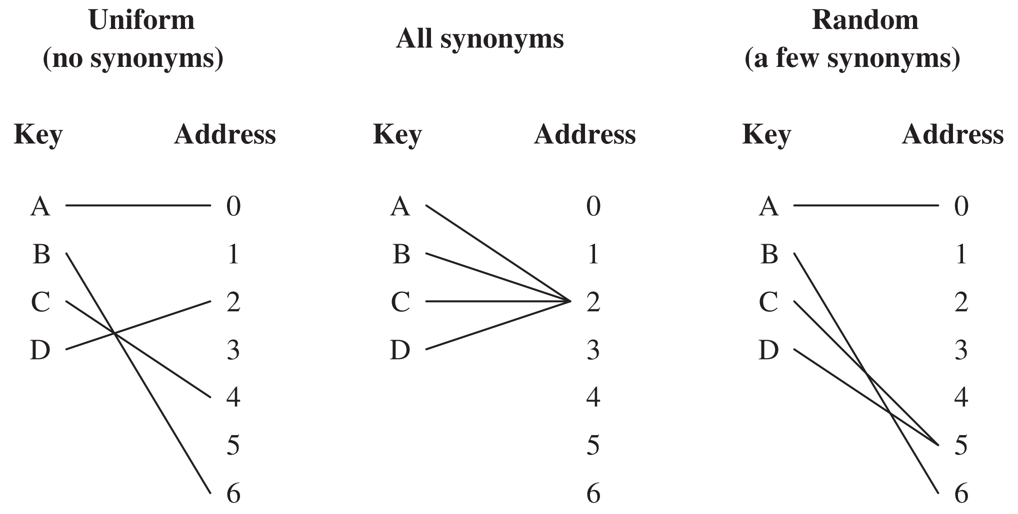
A Simple Hash Function

To compute this hash function, apply 3 steps :

Step 1 : transform the key into a number.
Step 2 : fold and add (chop off pieces of the number and add them together).
Step 3 : divide by the size of the address space (preferably a prime number).

Distribution of Records among Addresses

There are 3 possibilities :



Uniform distributions are extremely rare.
Random distributions are acceptable and more easily obtainable.

Trying a **better-than-random** distribution, by preserving natural ordering among the keys :

- Examine keys for patterns.
- Fold parts of the key.
- Use prime number when dividing the key.

When it does not work, try **randomization**. You can using the following Hash functions:

- **Square the key and take the middle**:
  Ex: key = 453 $453^2 = 205209$
  Extract the middle = 52.

- **Radix transformation**:
  Transform the number into another base and then divide by the maximum address.
  Ex: Addresses from 0 to 99
  key = 453 in base 11 : 382
  hash address = 382 mod 99 = 85.