BICONSISTENCY AND HOMONYMY IN DISTRIBUTED SYSTEMS WITH EDGE SYMMETRY

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ABSTRACT. The unexpected impact that port *labeling* has on the performance of the overall system has been extensively studied; in particular the focus has been on the choices which yield global consistency properties known as *sense of direction*. The impact has been documented for a large variety of distributed tasks for a large variety of communication topologies (for a recent survey see [8]).

These studies and the evidence they provided have shown the need and importance of understanding the *nature* of sense of direction and, in general, of labeling consistency. This requires identifying and investigating basic factors, determining their properties, and studying their interaction.

In this paper we concentrate on systems with *edge symmetry*, that is systems where there is a bijection between the port numbers assigned at the two end of a communication link. This is typically the case in communication networks; e.g., the "left-right" labeling in rings, the "compass" (north-south, east-west) labelings in meshes and tori, the "dimensional" labeling in hypercubes, the "chordal" labeling in chordal rings, the "cartographic" labeling of plane-embeddings, etc .... For these systems we study two important properties, *biconsistency* and *homonymy*.

Keywords: Sense of Direction, Consistency, Port Labeling, Naming Schemes.

1. Introduction

There are some issues in the design of distributed systems whose relevance to the overall performance of the system is not immediately apparent, especially at the higher layers. One such issue is the choice of *port* numbers; i.e., how to call (or, label) the communication ports available at the system sites. The unexpected impact that port *labeling*

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has on the performance of the overall system has been extensively studied; in particular the focus has been on the choices which yield global consistency properties known as *sense of direction*. The impact has been documented for a large variety of distributed tasks for a large variety of communication topologies (for a recent survey see [8]).

These studies and the evidence they provided have shown the need and importance of understanding the *nature* of sense of direction and, in general, of labeling consistency. This requires identifying and investigating basic factors, determining their properties, and studying their interaction.

In this paper we concentrate on systems with *edge symmetry*, that is systems where there is a bijection between the port numbers assigned at the two end of a communication link. This is typically the case in communication networks; e.g., the "left-right" labeling in rings, the "compass" (north-south, east-west) labelings in meshes and tori, the "dimensional" labeling in hypercubes, the "chordal" labeling in chordal rings, the "cartographic" labeling of plane-embeddings, etc .... For these systems we study two important properties, *biconsistency* and *homonymy*.

Biconsistency is a form of consistency stronger than weak sense of direction: in addition to weak sense of direction, biconsistent systems also have *backward consistency* [6, 10], a property which is of particular relevance in systems with advanced communication capabilities [10]. To date, nothing is known on the relationship between biconsistency and edge-symmetry except the *negative* result of [10] showing that edge symmetry is neither necessary nor sufficient for biconsistency. In this paper, we explicitly focus on the study of biconsistency in systems with edge-symmetry and establish several interesting results on their relationship. In particular, we show that in systems with edge-symmetry, *weak sense of direction implies the existence of biconsistency*. We also prove the surprising result that there are distributed systems where every consistent coding is either biconsistent or decodable, but *not both*. The relationship between biconsistency and the number of distinct port names is studied next. Of particular interest are *minimal* labelings, especially for systems with regular communication architectures (e.g., see [3, 4, 9]). We prove that the interplay between minimality and biconsistency is *crucial* in that it separates regular systems with edge symmetry from those without. In fact, in regular graphs, without edge-symmetry there is *no* minimal biconsistency; with edge symmetry, *every* minimal consistency is biconsistent. Furthermore, the regular systems with minimal biconsistency are the Cayley graphs with Cayley labelings.

In distributed systems with weak sense of direction, the consistent coding provides a *naming scheme* which is truly local, distributed and decentralized. Its properties make it applicable in many areas, eg. the naming problem in distributed object systems [1]. The system is said to be *homonymous* if each system entity refers to itself using the same name (e.g., "I"). Homonymy is a clear advantage in that it simplifies the coding. We study the relationship between homonymy, consistency and edge symmetry. We prove the general result showing that in systems with edge symmetry, any form of consistency (e.g., weak sense of direction, backward consistency, biconsistency), if it exists, it also exists in a homonymous form. In particular, all minimal sense of direction are homonymous.

2. Definitions

2.1. **Labeled Graphs.**

We represent a distributed system by a labeled graph where nodes correspond to processors (processes etc.), edges correspond to communication links, and labels correspond to port numbers. Let $G = (V, E)$ be a simple undirected graph; let $E(x)$ denote the set of edges incident to node $x \in V$, and $deg(x) = |E(x)|$ the degree of x.

Given $G = (V, E)$ and a set Σ of labels, a local labeling function λ_x of $x \in V$ is a function $\lambda_x : E(x) \rightarrow \Sigma$ which associates a label $l \in \Sigma$ to each edge $e \in E(x)$; λ_x is a *local orientation* if it is injective.

A set $\lambda = \{\lambda_x : x \in V\}$ of local labelings will be called *labeling* of G, and by (G, λ) we shall denote the corresponding *(edge-)labeled graph.*

A labeling λ is a *local orientation* if λ_x is a local orientation for all x. It is *minimal* if it uses $deg(G) = max\{deg(x) : x \in V\}$ labels. It is *symmetric* if there exists a bijection $\psi : \Sigma \rightarrow \Sigma$ such that for each $\langle x, y\rangle \in E$, $\lambda_y(\langle y, x\rangle) = \psi(\lambda_x(\langle x, y\rangle))$; ψ will be called the edge-symmetry function. A a symmetric labeling is a *coloring* when the edge-symmetry function is the identity.

A *walk* π in G is a sequence of edges in which the endpoint of one edge is the starting point of the next edge. Let $P[x]$ denote the set of all the walks starting from $x \in V$, $P[x, y]$ the set of walks starting from $x \in V$ and ending in $y \in V$. Let $\Lambda_x : P[x] \rightarrow \Sigma^+$ and $\Lambda = \{\Lambda_x : x \in V\}$

denote the extension of λ_x and λ, respectively, from edges to walks; let $\Lambda[x] = \{\Lambda_x(\pi) : \pi \in P[x]\}$, and $\Lambda[x,y] = \{\Lambda_x(\pi) : \pi \in P[x,y]\}$.

2.2. **Sense of Direction.** Let (G,λ) be a labeled graph. Intuitively, (G,λ) is said to have *Sense of Direction* when it is possible to understand, from the labels associated to the edges, whether different walks *originating* from any given node end in the same node or in different ones (see [7]). It is said to have *Backward Sense of Direction* when it is possible to understand, from the labels associated to the edges, whether different walks *terminating* in any given node start from the same node or in different ones (see, e.g. [10]).

More precisely, both types of sense of direction involves the existence of a coding and a decoding function.

Given (G,λ), a *coding function* $\mathbf{c}$ for λ is any function with domain Σ^+ and co-domain a set $\mathbf{N_c}$.

A coding function is *forward consistent* (or, simply, consistent) if walks originating from the same node are mapped to the same value (called *local name*) if and only if they end in the same node; that is, $\forall x,y,z \in V$, $\forall \pi_1 \in P[x,y]$, $\pi_2 \in P[x,z]$, $\mathbf{c}(\Lambda_x(\pi_1)) = \mathbf{c}(\Lambda_x(\pi_2)) \quad \Leftrightarrow \quad y = z$.

A coding function is *backward consistent* if $\forall x,y,z \in V$, $\forall \pi_1 \in P[x,z]$, $\pi_2 \in P[y,z]$ $\mathbf{c}(\Lambda_x(\pi_1)) = \mathbf{c}(\Lambda_y(\pi_2)) \Leftrightarrow x = y$. In other words, sequences of labels of walks *terminating* in the same node are mapped to the same value iff they start from the same node.

Note that the difference between forward and backward consistency is of "viewpoint". In the latter, the focus is *forwards*, on the sequence of labels on walks leaving from a given node; in the former the focus is *backwards* on the sequences of labels on walks which terminate at a given node.

When a coding function $\mathbf{c}$ is both forward and backward consistent, we will say that it is *biconsistent.*

We say that a coding function $\mathbf{c}$ is *homonymous* when, $\forall x,y \in V$, $\pi_1 \in P[x,x]$,$\pi_2 \in P[y,y]$: $\mathbf{c}(\Lambda_x(\pi_1)) = \mathbf{c}(\Lambda_y(\pi_2))$.

Definition 1. *(*$\mathbf{WSD}^{+(-)}$*)*
A system (G,λ)*, has* weak (backward) sense of direction *if and only if there exists a (backward) consistent coding function* $\mathbf{c}$*.*

We shall also say that $\mathbf{c}$ is a $\mathbf{WSD}^{+(-)}$.

A *decoding function* $\mathbf{d}$ for $\mathbf{c}$ is a function that, given a label and a coding of a string (sequence of labels), returns the coding of the concatenation of the label and the string.

More precisely, given a coding function $\mathbf{c}$, a (forward) decoding function for $\mathbf{c}$ is a function $\mathbf{d} : \Sigma \times \mathbf{N_c} \to \mathbf{N_c}$ such that $\forall \langle x, y\rangle \in E(x)$ and $\pi \in P[y, z]$, $\mathbf{d}(\lambda_x(\langle x, y\rangle), \mathbf{c}(\Lambda_y(\pi)) = \mathbf{c}(\lambda_x(\langle x, y\rangle) \cdot \Lambda_y(\pi))$, where $\cdot$ is the concatenation operator. In this case, we also say that $\mathbf{c}$ is *forward decodable* (or, simply, decodable).

Given a coding function $\mathbf{c}$, a *backward decoding function* for $\mathbf{c}$ is a function $\mathbf{b} : \Sigma \times \mathbf{N_c} \to \mathbf{N_c}$ such that $\forall \pi \in P[x, y]$, $\langle y, z\rangle \in E(y)$, $\mathbf{b}(\lambda_y(\langle y, z\rangle), \mathbf{c}(\Lambda_x(\pi))) = \mathbf{c}(\Lambda_x(\pi) \cdot \lambda_y(\langle y, z\rangle))$. In this case, we also say that $\mathbf{c}$ is *backward decodable.*

Definition 2. *(*$\mathbf{SD}^+$*)*
A system (G, λ)*, has a* sense of direction $(\mathbf{c}, \mathbf{d})$ *if and only if* $\mathbf{c}$ *is a* $\mathbf{WSD}^+$*, and* $\mathbf{d}$ *is a decoding function for* $\mathbf{c}$*.*

Definition 3. *(*$\mathbf{SD}^-$*)*
A system (G, λ) *has* backward sense of direction $(\mathbf{c}, \mathbf{b})$ *iff* $\mathbf{c}$ *is a* $\mathbf{WSD}^-$ *and* $\mathbf{b}$ *is a backward decoding function for* $\mathbf{c}$*.*

Notice that $\mathbf{SD}^{+(-)}$ is a stronger notion than $\mathbf{WSD}^{+(-)}$; in fact there exist labeled graphs with $\mathbf{WSD}^{+(-)}$ but without $\mathbf{SD}^{+(-)}$ (see, for example, [2] and [10]).

The following definitions refer to systems with edge symmetry. Let $\mathbf{WSD}^+ = \{(G, \lambda, \mathbf{c}) : \mathbf{c}$ consistent $\}$, $\mathbf{SD}^+ = \{(G, \lambda, \mathbf{c}) : \mathbf{c}$ consistent and decodable $\}$, $\mathbf{WSD}^- = \{(G, \lambda, \mathbf{c}) : \mathbf{c}$ backward consistent $\}$, $\mathbf{SD}^- = \{(G, \lambda, \mathbf{c}) : \mathbf{c}$ backward consistent and backward decodable $\}$, $\mathbf{WSD}^\pm = \{(G, \lambda, \mathbf{c}) : \mathbf{c}$ biconsistent $\}$, $\mathbf{SD}^\pm = \{(G, \lambda, \mathbf{c}) : \mathbf{c}$ biconsistent and forward decodable $\}$, $\mathbf{W}^+ = \{(G, \lambda) : \exists \mathbf{WSD}(G, \lambda, \mathbf{c})\}$, $\mathbf{D}^+ = \{(G, \lambda) : \exists \mathbf{SD}(G, \lambda, \mathbf{c})\}$, $\mathbf{W}^- = \{(G, \lambda) : \exists \mathbf{WSD}^-(G, \lambda, \mathbf{c})\}$, $\mathbf{D}^- = \{(G, \lambda) : \exists \mathbf{SD}^-(G, \lambda, \mathbf{c})\}$, $\mathbf{W}^\pm = \{(G, \lambda) : \exists \mathbf{WSD}^\pm(G, \lambda, \mathbf{c})\}$, $X_H = \{(G, \lambda) : \exists X(G, \lambda, \mathbf{c})$ homonymous $\}$, where $X \in \{\mathbf{WSD}, \mathbf{SD}\}$.

3. Biconsistency

3.1. **General results.** In this section we study the relationship between (Backward) Sense of Direction and Biconsistency.

The following theorem shows that if (G, λ) has Weak Sense of Direction, then there exists a coding function with Backward consistency.

Theorem 1. [10] *($\mathbf{W} = \mathbf{W}^-$); ($\mathbf{D} = \mathbf{D}^-$)*
If $(G, \lambda, \mathbf{c})$ is a $\mathbf{WSD}^+$ with edge symmetry, then, there exists a coding function $\mathbf{c}'$ such that $(G, \lambda, \mathbf{c}')$ is a $\mathbf{WSD}^-$; if $\mathbf{c}$ has a consistent decoding, there exists a consistent backward decoding of $\mathbf{c}'$. Analogously, if $(G, \lambda, \mathbf{c})$ is a $\mathbf{WSD}^-$ with edge symmetry, then, there exists a coding function $\mathbf{c}'$ such that $(G, \lambda, \mathbf{c}')$ is a $\mathbf{WSD}^+$; if $\mathbf{c}$ has a consistent backward decoding, there exists a consistent decoding of $\mathbf{c}'$.

From Theorem 1, it follows that systems with edge symmetry either have both types of consistency or none.

Using Theorem 1 to obtain the two forms of consistency implies the use of two distinct coding functions, which requires a large use of resources in the system. Thus, an immediate question is whether there exists systems with biconsistent codings. The first step towards answering such a question comes from the following theorem which states that even in system with edge symmetry, there exist non-biconsistent coding functions.

Theorem 2. [10] *($\mathbf{WSD}^+ - \mathbf{WSD}^- \neq \emptyset$) ($\mathbf{WSD}^- - \mathbf{WSD}^+ \neq \emptyset$)*
Edge symmetry is not sufficient for a (backward) consistent coding function to be biconsistent.

Thus the previous theorem states that edge symmetry and one form of consistency is not sufficient for a coding to be biconsistent, and as the next result shows it is also not necessary.

Theorem 3. [10]
Edge symmetry is not necessary for the existence of a biconsistent coding function.

On the other hand, we now show that, when there is weak sense of direction, there exists also a biconsistent coding function.

Theorem 4. *($\mathbf{W}^+ = \mathbf{W}^- = \mathbf{W}^{\pm}$)*
In a system (G, λ) with edge symmetry and $\mathbf{WSD}^{+(-)}$ there exists also a biconsistent $\mathbf{WSD}^{+(-)}$.

Proof. Let $\overline{\mathbf{c}}$ be a $\mathbf{WSD}$ in (G, λ), we will show how to construct a $\mathbf{WSD}$ $\mathbf{c}$ which is biconsistent. We define the equivalence relation R between strings on the labeling alphabet in the following way: $\forall \alpha, \beta \in \Sigma^+ : \alpha R \beta \Leftrightarrow \exists x, y \in V, \pi_1, \pi_2 \in P[x, y] : \Lambda_x(\pi_1) = \alpha \wedge \Lambda_x(\pi_2) = \beta$. Let R^+ be the transitive closure of R. Let $\mathbf{c}(\alpha) = [\alpha]_{R^+}$.

Claim 1. 1. $\forall \mathbf{c}' \in \mathbf{SD}, \alpha, \beta \in \Sigma^+ : \mathbf{c}(\alpha) = \mathbf{c}(\beta) \Rightarrow \mathbf{c}'(\alpha) = \mathbf{c}'(\beta)$.
2. $\forall \alpha, \beta : \mathbf{c}(\alpha) = \mathbf{c}(\beta) \Leftrightarrow \mathbf{c}(\psi(\alpha)) = \mathbf{c}(\psi(\beta))$
3. $[\psi(\alpha)]_{R^+} = \psi([\alpha]_{R^+})$.

We now prove that $\mathbf{c}$ is a consistent coding function. Let $x, y, z \in V, \pi_1 \in P[x,y] \pi_2 \in P[x,z] : \Lambda_x(\pi_1) = \alpha \wedge \Lambda_x(\pi_2) = \beta$, then we have to prove that $\mathbf{c}(\alpha) = \mathbf{c}(\beta)$ if and only if $y = z$. By property 1.1, $\mathbf{c}(\alpha) = \mathbf{c}(\beta)$ implies $\overline{\mathbf{c}}(\alpha) = \overline{\mathbf{c}}(\beta)$ and by consistence of $\overline{\mathbf{c}}$ it follows that $y = z$. If $y = z$ then by definition of R it holds that $\alpha R \beta$ and then $[\alpha]_{R^+} = [\beta]_{R^+}$ that is $\mathbf{c}(\alpha) = \mathbf{c}(\beta)$.
We now prove that $\mathbf{c}$ is backward consistent. By contradiction suppose that $\mathbf{c}$ is not consistent. Then there exists x, y, z and $\pi_1 \in P[x,z], \pi_2 \in P[y,z]$ such that it is not true that $\mathbf{c}(\Lambda_x(\pi_1)) = \mathbf{c}(\Lambda(\pi_2)) \Leftrightarrow x = y$. By property 1.2, this implies that there exists $x, y, z, \pi'_1 \in P[z,x], \pi'_2 \in P[z,y] : \Lambda_z(\pi'_1) = \psi(\alpha) \wedge \Lambda_z(\pi'_2) = \psi(\beta)$ and $\mathbf{c}(\psi(\alpha)) = \mathbf{c}(\psi(\beta)) \Leftrightarrow x = y$ is false. Contradicting the consistency of $\mathbf{c}$. □

The presence of biconsistency guaranteed by the above theorem is automatically true only for weak sense of direction. This property, unfortunately, does not extend to $\mathbf{SD}$, as the following theorem shows.

Theorem 5. *($\mathbf{D}^- \not\subset \mathbf{D}^+$)*
Sense of direction in a system with edge symmetry is not sufficient for the existence of a biconsistent coding function.

Proof. Consider the labeled graph of Figure 1. It is easy to verify that there exists a sense of direction for such a system. Let $(\mathbf{c}, \mathbf{d})$ be a $\mathbf{SD}$ and suppose, by contradiction, that $\mathbf{c}$ is backward consistent.
By definition of coding function at node x we have that $\mathbf{c}(df) = \mathbf{c}(be)$, it follows that: $\mathbf{d}(a, \mathbf{c}(be)) = \mathbf{d}(a, \mathbf{c}(df)) = \mathbf{c}(c)$, which means that $\mathbf{c}(abe) = \mathbf{c}(c)$ contradicting the hypothesis of backward consistency at node y. □

As corollary of the previous result we have the following theorem.

Theorem 6. *($\mathbf{D}^{\pm} \subset \mathbf{D}^{+(-)}$)*
Sense of direction in a system with edge symmetry, is not sufficient for the existence of a biconsistent sense of direction.

3.2. **Biconsistency and Minimality in Regular Networks.** In this section we ask some questions about the relationship between the set of labels used in (G, λ) and the existence of biconsistency. In particular we

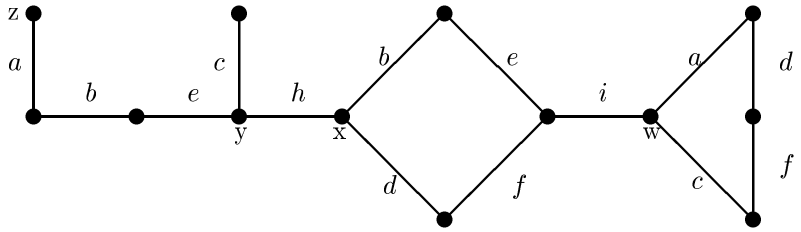


FIGURE 1. The graph of Theorem 5.

focus on the number $\kappa(G,\lambda) = |\Sigma|$ of labels used in (G,λ). A general bound on κ is the following, where $deg(G)$ and $e(G)$ denote the maximum degree and the number of edges in G, respectively:

Lemma 1. *If (G,λ) has (backward) consistency then $deg(G) \leq \kappa(G,\lambda) \leq 2e(G)$*

If $\kappa(G,\lambda) = deg(G)$, the labeling is said to be *minimal* and so is called any consistency property existing in (G,λ). So, consistency in minimally labeled systems is called *minimal (weak) sense of direction* (and denoted by $m(\mathbf{W})\mathbf{SD}$); similarly we can define the backward analogues ($m(\mathbf{W})\mathbf{SD}^-$).

We consider the relationship between minimality and biconsistency in regular networks. First observe that the requirement of minimality does not make edge symmetry necessary for either form of consistency:

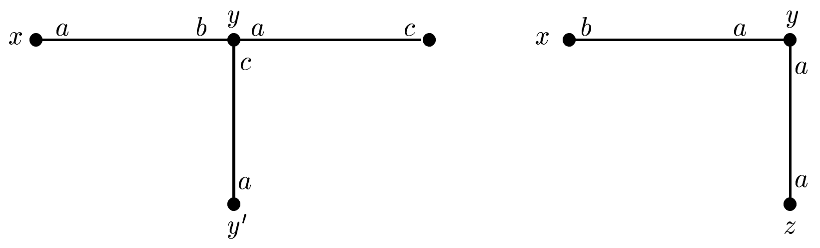
Lemma 2. *Edge symmetry is not necessary for having $m\mathbf{WSD}$ or $m\mathbf{WSD}^-$ in regular graphs.*

Proof. The non necessity for $m\mathbf{WSD}$ has been shown in [4]; the symmetric result in [10]. $\square$

However, unlike the general case (see Theorem 3), edge symmetry is necessary for biconsistency if also minimality is required.

Theorem 7. *Edge symmetry is necessary for having biconsistency in regular graphs with minimal labeling.*

Proof. By contradiction. Let (G,λ) have both a consistent coding $\mathbf{c}$ and a backward consistent coding $\mathbf{c_b}$ functions; but let λ not have edge symmetry. Since there is not edge symmetry, the graph is regular, and λ



$a)$ $b)$

FIGURE 2. Proof of Theorem 7.

is a local orientation, there must exist two edges $\langle x,y\rangle$, $\langle y,z\rangle$ such that $\lambda_x(\langle x,y\rangle)=a$, $\lambda_y(\langle y,x\rangle)=b$, $\lambda_y(\langle y,z\rangle)=a$, $\lambda_z(\langle z,y\rangle)=c$, with $b\neq c$. There are two possible situations depending on whether $a\neq c$ or $a=c$.
Let $a=c$. By definition of backward consistent coding we should have that: $\mathbf{c_b}(\lambda_x(\langle x,y\rangle))\neq\mathbf{c_b}(\lambda_z(\langle z,y\rangle))$, but $\lambda_x(\langle x,y\rangle)=\lambda_z(\langle z,y\rangle)$ (see Figure 2 b)) and the contradiction follows.
Let now $a\neq c$. By definition of consistent coding function, we have that $\mathbf{c}(\Lambda_y(\langle y,z\rangle\langle z,y\rangle\langle y,z\rangle))=\mathbf{c}(\lambda_y(\langle y,z\rangle))$, that is $\mathbf{c}(aca)=\mathbf{c}(a)$.
Since the graph is regular and there is local orientation, there must exist a path labeled aca starting from any vertex. Let $\langle x,y\rangle\langle y,y'\rangle\langle y',y''\rangle$ be such a path starting from x. Since $\mathbf{c}(aca)=\mathbf{c}(a)$, it follows that $y''=y$ (see Figure 2 a)). By definition of backward consistent coding we have that: $\mathbf{c_b}(\lambda_x(\langle x,y\rangle))\neq\mathbf{c_b}(\lambda_{y'}(\langle y',y\rangle))$. But $\lambda_x(\langle x,y\rangle)=\lambda_{y'}(\langle y',y\rangle)$ and the contradiction follows. □

Hence, we do need edge-symmetry for the simultaneous existence of both types of consistency. Let the system have edge symmetry. From Theorem 1, it follows that if the system has any form of minimal consistency, it has both. We can actually show that in these systems, any (backward) consistent coding is biconsistent.

Theorem 8. *($m\mathbf{W}^{\pm}=m\mathbf{W}^{+}=m\mathbf{W}^{-}$)*
In a regular labeled graph (G,λ) with edge symmetry, any $m\mathbf{WSD}^{+}$ ($m\mathbf{WSD}^{-}$) is also biconsistent.

Proof. Let $\mathbf{c}$ be a consistent coding function for (G, λ). By contradiction, let us assume that $\mathbf{c}$ does not have backward consistency which means that there exist two paths $\pi_1 \in P[x, z], \pi_2 \in P[y, z]$ such that: $i)$ either $\mathbf{c}(\Lambda_x(\pi_1)) = \mathbf{c}(\Lambda_y(\pi_2))$ and $x \neq y$ or $ii)$ $\mathbf{c}(\Lambda_x(\pi_1)) \neq \mathbf{c}(\Lambda_y(\pi_2))$ and $x = y$.
Let us consider case $i)$. Since the graph is regular and the labeling minimal there must exist a path $\pi_3 \in P[x]$ such that $\Lambda_x(\pi_3) = \Lambda_y(\pi_2)$. Thus, $\mathbf{c}(\Lambda_x(\pi_1)) = \mathbf{c}(\Lambda_y(\pi_3))$ which implies that $\pi_3 \in P[x, z]$. This is a contradiction because it violates local orientation in z.
Consider now case $ii)$. This case violates the definition of forward consistency in x.
The vice-versa is analogous. □

Thus, with edge symmetry, the class of regular graphs with $m\mathbf{WSD}^-$ coincides with the class of edge-symmetric regular graphs with $m\mathbf{WSD}$. Using the characterization of the latter class by [3, 9], it follows

Theorem 9. *A regular labeled graph (G, λ) with edge symmetry, has $m\mathbf{WSD}^-$ iff it is a Cayley graph with Cayley labeling.*

In other words, if there is edge symmetry, Cayley graphs with Cayley labelings are the only regular graphs with minimal backward sense of direction *and/or* minimal sense of direction.

Corollary 1. *($m\mathbf{W}^\pm = m\mathbf{D}^\pm$)*
In a regular labeled graph with edge symmetry, any minimal and biconsistent weak sense of direction is a sense of direction.

The situation is quite different in absence of edge symmetry; in this case, at most one type of minimal consistency can exist:

Lemma 3. *Without edge symmetry and restricted to regular graphs, $m\mathbf{W}^+ \cap m\mathbf{W}^- = \emptyset$*

Proof. By Theorem 7. □

3.3. **Biconsistency and Minimality Arbitrary networks.** We now study the relationship between minimality and biconsistency in arbitrary topologies.

Theorem 10. *($m\mathbf{WSD}^+ \not\subset m\mathbf{WSD}^-$)*
Edge Symmetry in a minimal **WSD** *is not sufficient for backward consistency.*

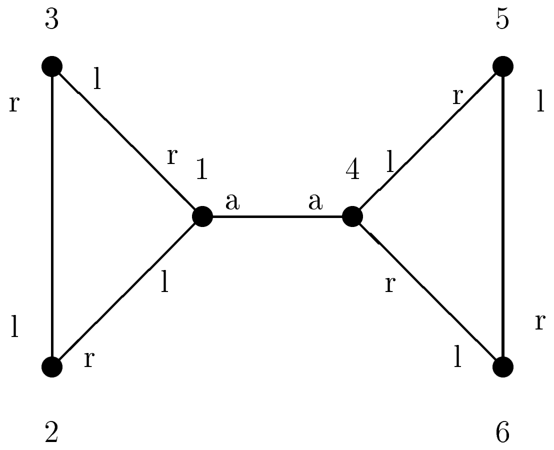


FIGURE 3. A minimal labeled graph with a consistent coding c that is not backward consistent.

Proof. Consider the labeled graph of Figure 3.3.

For each $\alpha \in \{r, l, a\}^*$, let $\mu(\alpha) \in \{r, a\}^*$ be the string obtained from α applying as many time as possible one of the following simplification rules: $rl \to \epsilon$, $lr \to \epsilon$, $aa \to \epsilon$, $rrr \to \epsilon$, $lll \to \epsilon$, $l \to rr$. The first three rules eliminates all the cycles of length 2, the following two rules eliminate all the cycles of length 3. The last rule rewrites the l with two r. Let $A = \{\alpha : \exists v \in V \wedge \exists \pi \in P[v] : \Lambda_v(\pi) = \alpha\}$, and let $\mu(A) = \{\mu(\alpha) : \alpha \in A\}$. It can be proved that $\mu(A) = r^{2*}(\cdot a \cdot r^{2*})^?$ where r^{2*} means at most 2 repetitions of the symbol r (can be 0 repetitions). $(x)^?$ means 0 or 1 repetition of x. Let a coding function $\mathbf{c}$ be defined as follows:

$$\mathbf{c}(\alpha) = \begin{cases} (\mu(\alpha), 0) & \text{if } \mu(\alpha) \in r^{2*} \\ (\beta, 1)) & \text{if } \mu(\alpha) = r^{2*} a \beta \end{cases}$$

For each node v let $\overline{v}$ be the node on the same ring with degree 3 (i.e. $\overline{2} = \overline{3} = \overline{1} = 1$ and $\overline{4} = \overline{5} = \overline{6} = 4$. It can be proved the following property:

Claim 2. 1. $\forall x, y, \pi \in P[x, y]$:
$(\mathbf{c}(\Lambda_x(\pi)) = \mathbf{c}(\gamma, 0) \Leftrightarrow \overline{x} = \overline{y} \wedge \exists \pi' \in P[x, y] \wedge \Lambda_x(\pi') = \gamma)$.

2. $\forall x, y, \pi \in P[x,y]$:
$(\mathbf{c}(\Lambda_x(\pi)) = \mathbf{c}(\gamma, 1) \Leftrightarrow \overline{x} \neq \overline{y} \wedge \exists \pi' \in P[\overline{y}, y] \wedge \Lambda_{\overline{y}}(\pi') = \gamma)$.

It is easy to see that $\mathbf{c}$ is a consistent coding function. However, $\mathbf{c}$ is not backward consistent. Indeed, since ra is the label of the path from 5 to 1, and a is the label of the path from 4 to 1, it follows that $\mathbf{c}(ra) \neq \mathbf{c}(a)$. On the other hand, $\mathbf{c}(ra) = (\epsilon, 1) = \mathbf{c}(a)$ yielding a contradiction. □

3.4. **Number of Labels and biconsistency.** In this section we ask under what conditions a (large) value $\kappa(G, \lambda)$ is sufficient for backward consistency.

If we are only interested in backward sense of direction, we can actually relax this condition. In fact, it is sufficient that the labels used by each node are different from the ones used by the other nodes, for the system to have $\mathbf{SD}^-$ (but not necessarily **SD**); see, for example, Figure 4. Let $\Sigma_x = \{\lambda_x(x,y) : y \in E(x)\}$.

Theorem 11. *If, in a system (G, λ) with edge symmetry, $\Sigma_x \cap \Sigma_y = \emptyset$ for all $x \neq y$, then (G, λ) has backward sense of direction.*

Proof. For each $a \in \Sigma$, only one node x uses a: let $\sigma(a) = \Sigma_x$.

Consider the following backward coding function. $\forall \pi \in P[x_0]$, $\pi = \langle x_0, x_1 \rangle \ldots \langle x_{m-1}, x_m \rangle$: $c_b(\Lambda_x(\pi)) = \sigma(\lambda_{x_0}(\langle x_0, x_1 \rangle))$.
Let $\pi_1 \in P[x,z]$, $\pi_2 \in P[y,z]$; $\pi_1 = \langle x, x_1 \rangle \ldots \langle x_h, z \rangle$, $\pi_2 = \langle y, y_1 \rangle \ldots \langle y_k, z \rangle$. We have that $c_b(\Lambda_x(\pi_1)) = \sigma(\lambda_x(\langle x, x_1 \rangle))$; and $c_b(\Lambda_y(\pi_2)) = \sigma(\lambda_y(\langle y, y_1 \rangle))$.

Case $x = y$. By definition of σ, we have that $\sigma(\lambda_x(\langle x, x_1 \rangle)) = \sigma(\lambda_y(\langle y, y_1 \rangle))$, thus, $c_b(\Lambda_x(\pi_1)) = c_b(\Lambda_y(\pi_2))$.

Case $x \neq y$. Since, by hypothesis, all the labels used by x are different from all the labels used by y, we have $\sigma(\lambda_x(\langle x, x_1 \rangle)) \neq \sigma(\lambda_y(\langle y, y_1 \rangle))$ and $c_b(\Lambda_x(\pi_1)) \neq c_b(\Lambda_y(\pi_2))$.
It is easy to see that the corresponding backward consistent decoding function is the following:
$\forall \pi \in P[x_0]$, $\pi = \langle x_0, x_1 \rangle \ldots \langle x_{m-1}, x_m \rangle$, $\forall \langle x_m, y \rangle \in E(x_m)$:
$d_b(\lambda_{x_m}(\langle x_m, y \rangle), c_b(\Lambda_{x_0}(\pi))) = c_b(\Lambda_{x_0}(\pi))$. □

Analogously, when the labels used for the edges terminating in x are different from the ones used for edges terminating in any $y \neq x$, then the system has **SD**. Let $\Sigma_x^- = \{\Sigma_y(y,x) : y \in E(x)\}$.

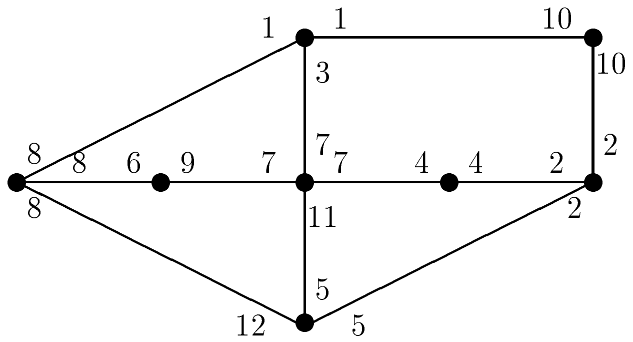


FIGURE 4. $\Sigma_x \cap \Sigma_y = \emptyset$.

Theorem 12. *If, in a system (G, λ) with edge symmetry, $\Sigma_x^- \cap \Sigma_y^- = \emptyset$ for all $x \neq y$, then (G, λ) has sense of direction.*

As a corollary, we have that when $\kappa(G, \lambda) = 2e(G)$ then the system has both **SD** and **SD**$^-$ without further conditions.

Corollary 2. *If, in a system (G, λ) with edge symmetry, $\kappa(G, \lambda) = 2e(G)$, then (G, λ) has a biconsistent sense of direction.*

Basically, this numerical requirement is equivalent to having all edges labeled with unique labels.

4. HOMONYMY

Another way to reduce the complexity of a coding function is requiring that all the cycles are coded into the same value, this equivalent to the situation where each system entity refers to itself using the same name.

With the following theorem we show that if any form of Sense of Direction is present in a labeled graph, then the same form of Sense of Direction plus Homonymy is also present. Note that the following result preserve the biconsistency.

Lemma 4. [11]
Let (G, λ) be a system with edge symmetry. If there exist $x, y, z \in V$, $y \neq$

z and $\pi_1 \in P[x,x]$, $\pi_2 \in P[y,z]$ such that $\Lambda_x(\pi_1) = \Lambda_y(\pi_2)$ then there does not exist any consistent coding function for (G,λ).

Theorem 13. *($X = X_H$, $X = \{\mathbf{W}^*, \mathbf{D}^*\}$)*
With edge symmetry, a system (G,λ) with (backward) Sense of Direction, has also a (backward) Sense of Direction with homonymy.

Proof. Let $\mathbf{c} : \Sigma^+ \to \mathbf{N_c}$ and let $\bigcirc$ be a new element (i.e. $\bigcirc \notin \mathbf{N_c}$) and $\overline{\mathbf{c}} : \Sigma \to^+ \mathbf{N_c} \cup \{\bigcirc\}$ s.t.

$$\overline{\mathbf{c}}(w) = \begin{cases} \bigcirc & \text{if } \exists x, \pi \in P[x,x] \wedge \Lambda_x(\pi) = w \\ \mathbf{c}(w) & \text{otherwise} \end{cases}$$

The sequences of labels of any cycle are mapped by $\overline{\mathbf{c}}$ to the symbol $\bigcirc$, then $\overline{\mathbf{c}}$ is homonymous.

Note: by the previous lemma, if $\pi \in P[x,y] \wedge x \neq y$, then $\overline{\mathbf{c}}(\Lambda_x(\pi)) = \mathbf{c}(\Lambda_x(\pi))$.

We will first show that $\overline{\mathbf{c}}$ is consistent. For every $x, y, z \in V, \pi_1 \in P[x,y], \pi_2 \in P[x,z]$ only four cases are possible:

1. $x = y = z$; thus $\pi_1, \pi_2 \in P[x,x]$ which implies $\bigcirc = \overline{\mathbf{c}}(\Lambda_x(\pi_1)) = \overline{\mathbf{c}}(\Lambda_x(\pi_2)) = \bigcirc$; that is, $\overline{\mathbf{c}}(\Lambda_x(\pi_1)) = \overline{\mathbf{c}}(\Lambda_x(\pi_2)) \Leftrightarrow y = z$.
2. $x = y \neq z$; thus $\pi_1 \in P[x,x]$ and $\pi_2 \in P[x,z]$ which imply $\bigcirc = \overline{\mathbf{c}}(\Lambda_x(\pi_1)) \neq \overline{\mathbf{c}}(\Lambda_x(\pi_2)) = \mathbf{c}(\lambda_x(\pi_2)) \in \Lambda$. That is, $\overline{\mathbf{c}}(\Lambda_x(\pi_1)) = \overline{\mathbf{c}}(\Lambda_y(\pi_2)) \Leftrightarrow y = z$.
3. $x = z \neq y$; As in case 2 above switching y and z.
4. $x \neq y$ and $x \neq z$; thus, $\mathbf{c}(\Lambda_x(\pi_1)) = \overline{\mathbf{c}}(\Lambda_x(\pi_1))$ and $\mathbf{c}(\Lambda_x(\pi_2)) = \overline{\mathbf{c}}(\Lambda_x(\pi_2))$. By consistency of $\mathbf{c}$, $\mathbf{c}(\Lambda_x(\pi_1)) = \mathbf{c}(\Lambda_x(\pi_2)) \Leftrightarrow y = z$.

Thus, we have proved that there exists $\overline{\mathbf{c}}$ in $\mathbf{WSD} \cap \mathbf{H}$. It is easy to see that if c is backward consistent, then the above $\overline{\mathbf{c}}$ is also backward consistent. A symmetric proof works also for this case. Thus, $\exists \mathbf{c} \in \mathbf{WSD}^-$ implies $\exists \overline{\mathbf{c}} \in \mathbf{WSD}^- \cap \mathbf{H}$, moreover, $\exists \mathbf{c} \in \mathbf{WSD} \cap \mathbf{WSD}^-$ implies $\exists \overline{\mathbf{c}} \in \mathbf{WSD} \cap \mathbf{WSD}^- \cap \mathbf{H}$.

Now we will prove that if $(\mathbf{c}, \mathbf{d}) \in \mathbf{SD}$, then $(\overline{\mathbf{c}}, \overline{\mathbf{d}}) \in \mathbf{SD} \cap \mathbf{H}$ where $\overline{\mathbf{c}}$ is defined as before and $\overline{\mathbf{d}} : \Sigma \times (\mathbf{N_c} \cup \{\bigcirc\}) \to \mathbf{N_c} \cup \{\bigcirc\}$ is defined as follows:

$$\overline{\mathbf{d}}(a,n) = \begin{cases} \overline{\mathbf{c}}(a) & \text{if } n = \bigcirc \\ \bigcirc & \text{if } \exists x, y, \langle x,y \rangle \pi \in P[x,x] \wedge \\ & \wedge \Lambda_x(\langle x,y \rangle \pi) = a \cdot w \wedge \mathbf{c}(w) = n \\ \mathbf{d}(a,n) & \text{otherwise.} \end{cases}$$

Claim 3. *For each $a \in \Sigma$ and each $n \in \mathbf{N}$: if $\exists x', y', z' : x' \neq z' \wedge \langle x', y'\rangle\pi' \in P[x', z'] \wedge \Lambda_{x'}(\langle x', y'\rangle\pi') = a \cdot w' \wedge \mathbf{c}(w') = n$, then $\not\exists x, y, \langle x, y\rangle\pi \in P[x, x] : \Lambda_x(\langle y, z\rangle\pi) = a \cdot w \wedge \mathbf{c}(w) = n$.*

Proof. (**of Claim**)
Let $x, y \in V$ and $\langle y, z\rangle\pi \in P[x, x]$ be such that $\Lambda_x(\langle y, z\rangle\pi) = a \cdot w$; and let $x', y', z' \in V(x' \neq z')$ and $\langle x', y'\rangle\pi' \in P[x', z']$ be such that $\Lambda_{x'}(\langle x', y'\rangle) = a \cdot w' \wedge \mathbf{c}(w') = n$. Since $\langle y, x\rangle \in P[y, x]$ and $\pi \in P[y, x]$ by consistency of $\mathbf{c}$, $\mathbf{c}(\lambda_y(\langle y, x\rangle)) = \mathbf{c}(\psi(a)) = \mathbf{c}(\Lambda_y(\pi)) = \mathbf{c}(w)$. Since $\langle y', x'\rangle \in P[y', x']$ and $\pi' \in P[y', x']$, by consistency of $\mathbf{c}$ and since $x' \neq z'$, it follows that $\mathbf{c}(\lambda_{y'}(\langle y', x'\rangle)) = \mathbf{c}(\psi(a)) \neq \mathbf{c}(\Lambda_{y'}(\pi')) = \mathbf{c}(w') = n$. Thus $n \neq \mathbf{c}(w)$. □

We now prove that $\overline{\mathbf{d}}$ is a consistent decoding function for $\overline{\mathbf{c}}$. For every $x, y, z \in V, \langle x, y\rangle \in E, \pi \in P[y, z]$; there are only three possible cases:

1. $y = z$; thus, $\pi \in P[y, y]$ and, by definition , $\overline{\mathbf{c}}(\Lambda_y(\pi)) = \bigcirc$. Now, $\overline{\mathbf{d}}(\lambda_x(\langle x, y\rangle), \overline{\mathbf{c}}(\Lambda_y(\pi))) = \overline{\mathbf{d}}(\lambda_x(\langle x, y\rangle), \bigcirc) = \overline{\mathbf{c}}(\lambda_x(\langle x, y\rangle))$. Since $\langle x, y\rangle\pi \in P[x, y] \wedge \langle x, y\rangle \in P[x, y]$, by consistency of $\overline{\mathbf{c}}$, $\overline{\mathbf{c}}(\lambda_x(\langle x, y\rangle)) = \overline{\mathbf{c}}(\Lambda_x(\langle x, y\rangle\pi))$. Thus $\overline{\mathbf{d}}(\lambda_x(\langle x, y\rangle), \overline{\mathbf{c}}(\Lambda_y(\pi))) = \overline{\mathbf{c}}(\Lambda_x(\langle x, y\rangle\pi))$.
2. $x = z$; thus, $\langle x, y\rangle\pi \in P[x, x]$ and by definition of $\overline{\mathbf{d}}$, $\overline{\mathbf{d}}(\lambda_x(\langle x, y\rangle), \overline{\mathbf{c}}(\Lambda_y(\pi))) = \bigcirc$. By definition of $\overline{\mathbf{c}}$, $\overline{\mathbf{c}}(\Lambda_x(\langle x, y\rangle\pi)) = \bigcirc$.
3. $x \neq z \neq y$; thus, $\overline{\mathbf{c}}(\Lambda_y(\pi)) = \mathbf{c}(\Lambda_y(\pi))$ and, by Claim 3, $\overline{\mathbf{d}}(\lambda_x(\langle x, y\rangle), \overline{\mathbf{c}}(\Lambda_y(\pi))) = \mathbf{d}(\lambda_x(\langle x, y\rangle), \mathbf{c}(\Lambda_y(\pi)))$. Since d is a consistent decoding function for $\mathbf{c}$, then $\overline{\mathbf{d}}(\lambda_x(\langle x, y\rangle), \mathbf{c}(\Lambda_y(\pi))) = \mathbf{c}(\Lambda_x(\langle x, y\rangle\pi)) = \overline{\mathbf{c}}(\Lambda_x(\langle x, y\rangle\pi))$.

Thus $(\overline{\mathbf{c}}, \overline{\mathbf{d}})$ is a Sense of Direction.

In the symmetric way it can be proved that if $(\mathbf{c}, \mathbf{d})$ is a Backward Sense of Direction, then $(\overline{\mathbf{c}}, \overline{\mathbf{d}})$ is a Backward Sense of Direction. And also, if $(\mathbf{c}, \mathbf{d})$ is a Biconsistent Sense of Direction, then $(\overline{\mathbf{c}}, \overline{\mathbf{d}})$ is a Biconsistent Sense of Direction. □

The question is now to determine when a labeled graph has only homonymous coding functions. If λ is a coloring, then every $(\mathbf{W})\mathbf{SD}^+$ ($(\mathbf{W})\mathbf{SD}^-$) is homonymous as the following theorem shows.

Theorem 14. *In connected graphs with edge symmetry, all colorings* **SD** *are homonymous.*

Proof. Let $(\mathbf{c}, \mathbf{d})$ be a **SD** in (G, λ), where λ is a coloring. By contradiction. Suppose that the system is not homonymous; that is there exist u,$v \in V$, $\pi_1 \in P[u,u]$, $\pi_2 \in P[v,v]$, such that $\mathbf{c}(\Lambda_u(\pi_1)) = h \neq \mathbf{c}(\Lambda_v(\pi_2))$. Consider a path connecting u and v (such a path exists s-ince G is connected). Let $\pi = [u = x_0, x_1, \ldots x_i, \ldots, x_m = v]$ be the path. Let x_i and x_{i+1} such that $\exists \pi \in P[x_i, x_i] \wedge \exists \pi' \in P[x_{i+1}, x_{i+1}] \wedge \mathbf{c}(\Lambda_{x_i}(\pi)) = h \neq \mathbf{c}(\Lambda_{x_{i+1}}(\pi'))$. By consistency of $\mathbf{c}$, $\mathbf{c}(\Lambda_{x_i}(\pi)) = \mathbf{c}(\Lambda_{x_i}(\langle x_i, x_{i+1}\rangle\langle x_{i+1}, x_i\rangle))$, and $\mathbf{c}(\Lambda_{x_{i+1}}(\pi')) = \mathbf{c}(\Lambda_{x_{i+1}}(\langle x_{i+1}, x_i\rangle\langle x_i, x_{i+1}\rangle))$. But λ is a coloring, then $\Lambda_{x_i}(\langle x_i, x_{i+1}\rangle\langle x_{i+1}, x_i\rangle) = \Lambda_{x_{i+1}}(\langle x_{i+1}, x_i\rangle\langle x_i, x_{i+1}\rangle)$. Then, $\mathbf{c}(\Lambda_{x_i}(\pi))$ $= \mathbf{c}(\Lambda_{x_{x+1}}(\pi'))$. Contradiction. □

Moreover,

Theorem 15. *In connected graphs with edge symmetry, all minimal* **SD** *are homonymous.*

Proof. Let $v \in V$ a vertex with maximum degree (i.e. $deg(v) = deg(G)$). For each $\langle v, y\rangle \in E$ the path $\langle v, y\rangle\langle y, v\rangle$ belongs to $P[v,v]$, then by consistency of $\mathbf{c}$ it must be $\mathbf{c}(\Lambda_v(\langle v, y\rangle\langle y, v\rangle)) = \mathbf{c}(\Lambda_v(\langle v, z\rangle\langle z, v\rangle))$. This implies that for each $a, b \in \Sigma$, $\mathbf{c}(a \cdot \psi(a)) = \mathbf{c}(b \cdot \psi(b)) = h$ for some h. Thus for any node x, x is connected to the graph with at least one edge $\langle x, z\rangle$ and the label of $\lambda_x(\langle x, z\rangle) = \lambda_u(\langle v, y\rangle)$ for some y. Thus $\mathbf{c}(\Lambda_x(\langle x, z\rangle\langle x, z\rangle)) = \mathbf{c}(\Lambda_v(\langle v, y\rangle\langle y, v\rangle)) = \mathbf{c}(b \cdot \psi(b)) = h$. The consistency of c assure that $\mathbf{c}(\Lambda_x(\pi)) = h$ for each $\pi \in P[x,x]$. □

From Theorem 5 we know that **SD** is not sufficient for the existence of a biconsistent coding function. The next result gives a sufficient condition for an homonymous **SD** to be biconsistent.

Theorem 16. *Let* $(\mathbf{c}, \mathbf{d})$ *be an homonymous Sense of Direction in a system* (G, λ) *with edge symmetry. If* $\mathbf{c}$ *is backward decodable, then* $\mathbf{c}$ *is biconsistent.*

Proof. Let $(\mathbf{c}, \mathbf{d})$ be a Sense of Direction in (G, λ) and let $\mathbf{d}_b$ be a backward decoding function for $\mathbf{c}$. By contradiction, let $\exists x, y, z \in V$, $\pi_1 \in P[x,z]$, $\pi_2 \in P[y,z]$: $\mathbf{c}(\Lambda_x(\pi_1)) = \mathbf{c}(\Lambda_y(\pi_2))$ but $x \neq y$. Let $\Lambda_x(\pi_1) = \alpha$ and $\Lambda_y(\pi_2) = \beta$.
By definition of homonymy, we have that $\mathbf{c}(\alpha \cdot \Psi(\alpha)) = \mathbf{c}(\beta \cdot \Psi(\beta))$.
But, $\mathbf{d}_b(c(\alpha), \Psi(\alpha)) = \mathbf{d}_b(c(\beta), \Psi(\alpha))$ and thus, $\mathbf{c}(\alpha \cdot \Psi(\alpha)) = \mathbf{c}(\beta \cdot \Psi(\alpha))$.
By the consistency of $\mathbf{c}$, $\mathbf{c}(\beta \cdot \Psi(\alpha)) \neq \mathbf{c}(\beta \cdot \Psi(\beta))$; thus, $\mathbf{c}(\alpha \cdot \Psi(\alpha)) \neq \mathbf{c}(\beta \cdot \Psi(\beta))$ which contradicts the homonymy assumption. □

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